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Bending and Buckling Analysis of Porous 2D Functionally Graded Beams with Exponential Material Property Variation

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Abstract

The bending and buckling analysis of porous two-directional (2D) functionally graded (FG) beams was conducted using a higher-order shear deformation theory (HSDT). The introduction of exponential functions to depict changes in material properties is a novel approach in the static analysis of 2D FG beams. Three distinct porosity distribution functions were taken into account. The governing equations were formulated through the application of Lagrange's principle. During the numerical analysis, a finite element comprising two nodes and eight degrees of freedom (DOFs) was utilized. This choice facilitated accurate and efficient solutions, even for shorter beams, without the need for a shear correction factor. Notably, the obtained shear stresses aligned with actual values, registering as zero at both the top and bottom of the beam. The obtained results of the study were validated against findings reported in the literature. A parametric study was carried out to investigate the effects of porosity, porosity distributions, gradation parameters, slenderness, and boundary conditions on the non-dimensional deflections, stresses, critical buckling loads, and buckling mode shapes. It was found that both porosity and the distribution of porosity have noticeable effects on the static analysis of the beams.

Key Words: 2D functionally graded, HSDT, finite element, porous material

1. Introduction

Functionally Graded Materials (FGMs) are advanced engineered materials with tailored variations in composition, microstructure, and properties over their spatial dimensions. Unlike traditional homogeneous materials, FGMs exhibit a continuous transition of properties, allowing for seamless integration between distinct material phases. This unique design strategy enables FGMs to optimize performance, enhance structural integrity, and mitigate stress concentration, making them valuable for a wide range of applications in aerospace, energy, biomedical, and other industries. Hence, numerous researchers have explored the mechanical properties of FGMs through the utilization of analytical and numerical methods. Some of the papers on traditional (one dimensional – 1D) FG beams are summarized below.

Exact solutions for bending and free vibration of 1D FG beams resting on a Winkler–Pasternak elastic foundation was presented by Ying et al. [1] based on the theory of plain elasticity. Thai and Vo [2] presented analytical solutions for various higher-order shear deformation beam theories for bending and free vibration of 1D FG beams. The first-order shear deformation beam theory for static and free vibration of 1D FG rectangular simple supported beams was developed by Nyugen et al. [3] using an analytical approach. Aydogdu and Taskin [4] presented free vibration of simply supported 1D FG beams using a Navier type solution method. A closed-form solution of the Reddy beam theory was developed and applied to investigate the bending behaviour of straight and curved 1D FG beams by Ruocco and Reddy [5].

A new beam element based on first order shear deformation theory (FSDT) was developed by Chakraborty et al. [6] to study the thermoelastic behaviour of 1D FG beams structures. Kapuria et al. [7] investigated a third order zigzag theory for 1D FG beams based on a layered model in conjunction with the modified rule of mixtures for effective modulus of elasticity using FEM and the numerical results were validated through experiments for static and free vibration response. A five nodes and ten degree of freedoms (DOFs) finite element model based on the first-order shear deformation theory for free vibration and buckling of 1D FG beams was developed by Turan and Kahya [8]. Oyekoya et al. [9] presented Mindlin-type element and Reissner-type element for the buckling and vibration analysis of 1D DG composite structures using FEM. A static analysis of 1D FG single and sandwich beams by using an efficient 7DOFs quasi-3D hybrid type theory based on Hermite–Lagrangian finite element formulation was investigated by Yarasca et al [10].

In tandem with the progress made in the field of FGMs, researchers have also delved into the realm of two-dimensional (2D) FGMs, producing these materials and subsequently scrutinizing their

mechanical properties. Karamanli presented the static [11] and free vibration [12] behaviours of 2D FG sandwich beams subjected to various sets of boundary. The exact solution for bending analysis of 2D FG Timoshenko beams was investigated by Huang and Ouyang [13]. Huang [14] studied free vibration of non-uniform Timoshenko–Ehrenfest beams with arbitrary 2D FGMs using power series. Navier’s method based on the FSDT for bending analysis of 2D FG beams subjected to various sets of boundary conditions was presented Turan [15].

Karamanli and Vo [16] investigated the flexural behaviour of 2D FG microbeams subjected to uniformly distributed load with various boundary conditions using FEM. Elasto-static bending behaviour of 2D FG sandwich beams under various types of nonuniform distributed load was studied by Nyugen et al. [17]. Al-Zahrani et al. [18] investigated the free vibration of 2D FG plane stress strip using finite element method.

The inherent presence of pores in FGMs, arising from production processes, underscores the importance of investigating their mechanical properties. These pores can significantly influence crucial mechanical characteristics, such as strength, stiffness, and fracture resistance, ultimately impacting the material's overall performance. Therefore, a thorough examination of the mechanical properties of FGMs becomes imperative to comprehend the interplay between pore morphology and material behaviour, guiding the design and optimization of FGMs for enhanced structural integrity and functional efficiency.

Chen et al. [19] presented the elastic buckling and static bending analysis of shear deformable porous FG beams based on the Timoshenko beam theory using Ritz method. A new higher order shear deformation model was developed for static and free vibration analysis of functionally graded beams with considering porosities using Navier solution by Hanji et al. [20]. Nyugen et al. [21] presented a new two-variable shear deformation theory for bending, free vibration and buckling analysis of porous FG beams using Ritz method. The flexural vibration analysis of beams made of FGMs with various boundary conditions was considered using Chebyshev collocation method by Wattanasakulpong and Chaikittiratana [22]. Sayyad et al. [23] studied the static deformation and free vibration analysis of simply supported porous FG circular beams considering the effects of even and uneven porosities using Navier solution. A modified series solution for free vibration analyses of moderately thick porous FG deep curved and straight beams was investigated by Zhao et al. [24]. Shafiei et al. [25] presented the analysis on the vibration behaviour of the 2D FG nano and microbeams which are made of two kinds of porous materials based on Timoshenko beam theory. Chen et al. [26] investigated linear and nonlinear free vibration analysis of rotating porous 2D FG

micro-beam based on the Timoshenko beam theory, von Karman geometric nonlinearity assumption and the modified couple stress theory. A quasi-3D solution to develop a computational theoretical model to examine the buckling response of porous 2D beams exposed to variable axial load was presented by Sekkal et al. [27]. Fazzolari [28] investigated the free vibration and elastic stability behaviour of three-dimensional (3D) functionally graded sandwich beams featured by two different types of porosity, with arbitrary boundary conditions and resting on Winkler-Pasternak elastic foundations using the method of series expansion of displacement components.

Belarbi et al. [29] studied the bending and free vibration analysis of porous functionally graded sandwich plates with various porosity distributions using an efficient layer wise model and FEM. The size-dependent behaviours of metal foam microbeams with three different porosity distribution models were studied by Karamanli et al. [30] using FEM. Turan et al. [31] presented an analytical solution based on FSDT was performed for free vibration and buckling analysis of porous FG beams subjected to various boundary conditions using FE and artificial neural network methods. A new enhanced first-order beam element based on the neutral surface position was developed for bending analysis of the porous FG beams by Van Vinh et al. [32].

Karamanli and Vo [33] presented FE model for the structural behaviours of porous 2D microbeams based on a quasi-3D theory and the modified strain gradient theory (MSGT). Turan and Adiyaman presented a new higher-order finite element for the static [34] and free vibration [35] analysis of porous 2D FG beams subjected to various boundary conditions based on parabolic shear deformation theory (PSDT). The free vibration analysis of a porous 2D FG beams based on HSDT was investigated by Adiyaman [36] using FEM.

The authors' literature review reveals a notable lack of consideration for the Finite Element Method (FEM) among researchers studying the bending and buckling analysis of porous 2D beams. If HSDT is not employed in the FE solution, the resulting shear deformations deviate from the actual deformations, especially in short beams, even when a shear correction factor is introduced. However, a substantial portion of the existing studies have predominantly utilized classical theory or FSDT in their investigations. This paper explores the bending and buckling analysis of porous 2D beams, employing the principles of HSDT. The distinctive features of this study encompass the pioneering application of exponential modifications in the material properties of 2D FG beams, obviating the need for a shear correction factor, and considering the adverse influence of porosity as a product in formulations. Three distinct porosity distribution types were evaluated, employing a finite element model with two nodes and 8 degrees of freedom (DOFs) for the FE analysis. Following the

completion of convergence and verification studies, a parametric investigation was conducted to examine the impacts of porosity, material properties and geometric characteristics on the bending and buckling behaviours of porous 2D functionally graded beams.

2. Theoretical Formulation for Porous 2D Beams

2.1. Porous 2D Beams

Consider a two-directional FG beam of height h and length L as given in Fig. 1. The variation of material properties, denoted as $P(x, z)$, including Young's modulus $E(x, z)$, mass density $\rho(x, z)$, and Poisson ratio $\nu(x, z)$ in the beam can be expressed as follows,

$$P(x, z) = \mu(z) P_0 \exp\left(\frac{x}{L} p_x\right) \exp\left(\left(\frac{z}{h} + 0.5\right) p_z\right) \quad (1)$$

in which, the function $\mu(z)$ represents the porosity distribution and signifies the reduction caused by porosity on material properties; p_x and p_z denote the gradation parameters in x and z directions, respectively; and P_0 is the material property at the bottom-left corner $\left(0, -\frac{h}{2}\right)$ of the beam. It is assumed that Poisson's ratio remains uniform throughout the beam. In this study, three distribution functions [21] were examined, corresponding to *Type-H*, *Type-X*, and *Type-O*, as indicated below.

$$\mu_{1,E}(z) = \left(\frac{2}{\pi} \sqrt{1-e} - \frac{2}{\pi} + 1\right)^2, \quad \mu_{1,\rho}(z) = \left(\frac{2}{\pi} (1-e_m) - \frac{2}{\pi} + 1\right) \quad (2a,b)$$

$$\mu_{2,E}(z) = \left(1 - e \cos\left(\left|\frac{\pi}{h} z\right| - \frac{\pi}{2}\right)\right), \quad \mu_{2,\rho}(z) = \left(1 - e_m \cos\left(\left|\frac{\pi}{h} z\right| - \frac{\pi}{2}\right)\right) \quad (3a,b)$$

$$\mu_{3,E}(z) = \left(1 - e \cos\left(\frac{\pi}{h} z\right)\right), \quad \mu_{3,\rho}(z) = \left(1 - e_m \cos\left(\frac{\pi}{h} z\right)\right) \quad (4a,b)$$

In equations (2-4), E and ρ respectively represent the distribution functions for Young's modulus and mass density, while e and $e_m = 1 - \sqrt{1-e}$ are the porosity coefficients for the corresponding properties. The porosity distributions along the cross-section of the beam for Type-H (2), Type-X (3), and Type-O (4) are depicted in Fig. 2. As evident from the figure, porosities exhibit uniform distribution in Type-H, while they are concentrated at the upper and lower regions of the cross-section for Type-X, and at the midpoint for Type-O.

Fig. 3 gives the variation of Young's modulus through the beam in case of non-porous ($e=0$), Type-H, Type-X and Type-0 for various gradation parameters. Observing the graph, it becomes evident that E converges towards the value observed in the non-porous beam when the porosity distribution function approaches $\mu(z) \rightarrow 1$.

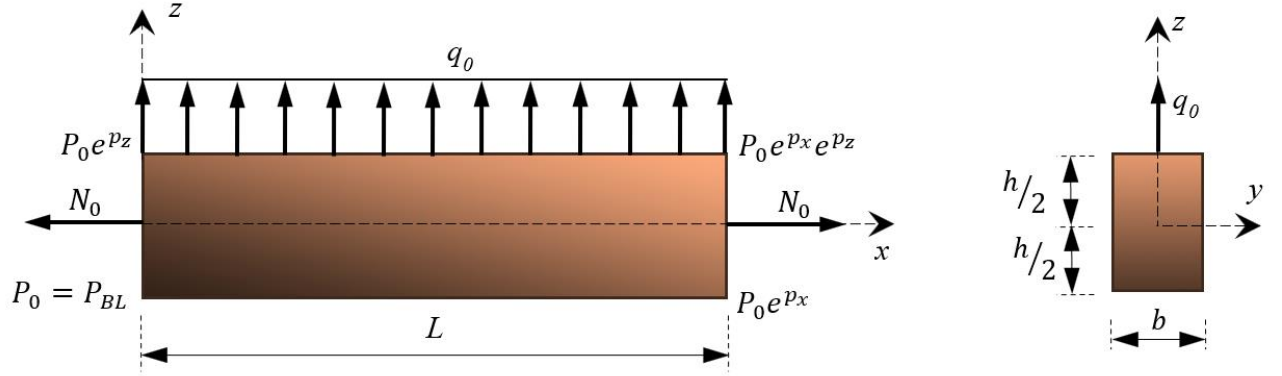


Fig. 1. The geometry and loading of porous 2D FG beam

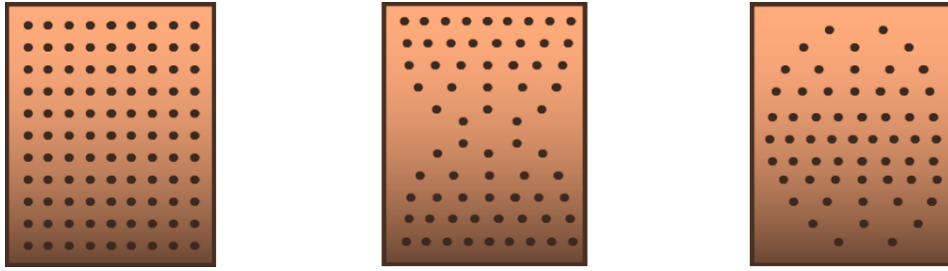


Fig. 2. The distribution of the porosity along the cross-section

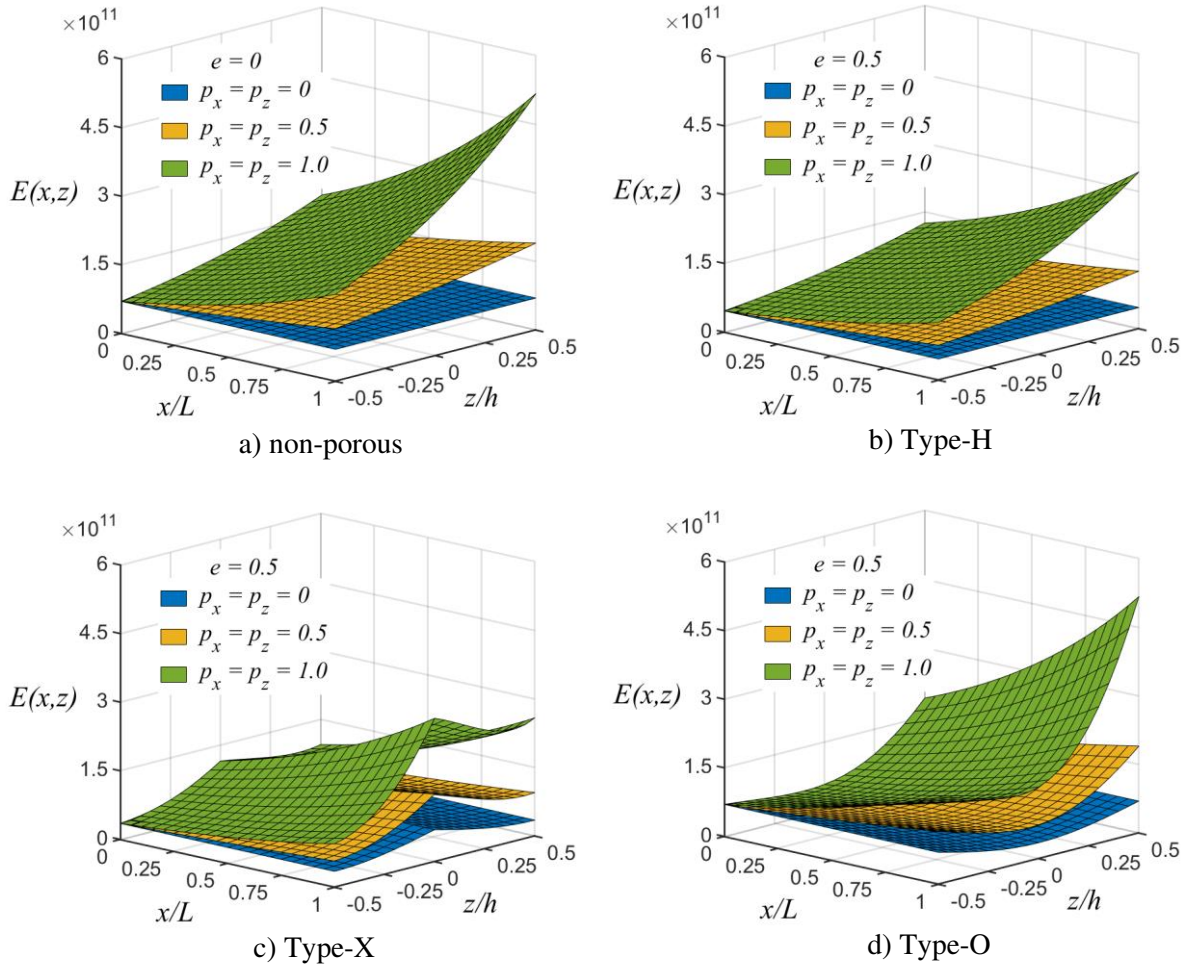


Fig. 3. The distribution of the porosity along the cross-section

2.2. Mathematical Formulation

The displacement field of the beam, in accordance with Reddy's third-order shear deformation theory that satisfies zero shear stress at the top and bottom of the beam, can be formulated as referenced in [49].

$$u(x, z) = u_0(x) - zw_{,x} + f(z)\phi, \quad w(x) = w_0(x), \quad f(z) = z - \frac{4z^3}{3h^2} \quad (5-7)$$

In equations (5-7), u and v represent the displacements along the x and y axes, respectively; u_0 and v_0 signify the displacements at the natural axis ($z = 0$); $w_{,x} = \frac{dw}{dx}$ and ϕ denote terms associated with rotations; and $f(z)$ is a third-order polynomial capturing higher-order deformations within the cross-section. The strain field can be expressed as,

$$\varepsilon_{xx} = \frac{du_0(x)}{dz} - z \frac{dw_{,x}}{dx} + f(z) \frac{d\phi}{dx}, \quad \gamma_{xz} = \frac{df(z)}{dz} \phi \quad (8,9)$$

By considering the shear modulus as $G(x, z) = \frac{E(x, z)}{2(1-\nu)}$, the relationships between stress and shear can be described as,

$$\sigma_{xx} = E(x, z)\varepsilon_{xx}, \quad \tau_{xz} = G(x, z)\gamma_{xz} \quad (10,11)$$

The strain (U) and kinetic (K) energy of the beam can be expressed as follows.

$$U = \int_0^L \int_A (\sigma_{xx}\varepsilon_{xx} + \tau_{xz}\gamma_{xz}) dA dx, \quad K = \int_0^L \int_A \rho(z) (\dot{u}^2 + \dot{w}^2) dA dx \quad (12,13)$$

where, the dot ($\dot{}$) signifies differentiation with respect to time. The work (V) done by axial force N_0 and uniform distributed load q_0 can be formulated as given below.

$$V = \int_0^L N_0 (w_{,x})^2 dx + \int_0^L q_0 w_0 dx \quad (14)$$

The total energy (Π) of the beam can be obtained combining Eqs. (12-14) in the following form.

$$\Pi = U + V - K \quad (15)$$

2.3. Numerical Solution using FEM

The numerical solution for the porous 2D beam can be accomplished through the utilization of FEM.

Consider the higher-order beam element of length L_e with 2 nodes and 8 DOFs as shown in Fig. 4.

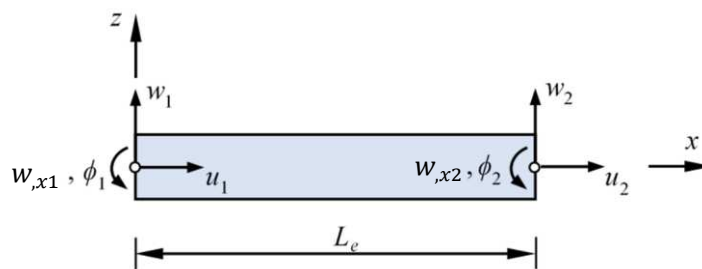


Fig. 4. The higher-order beam element with 2 nodes and 8 DOFs

By applying interpolation, the generalized nodal displacements for the displacement field can be derived as follows.

$$u_0(x) = \sum_{i=1}^2 \psi_{1i}(x) u_i, \quad w_0(x) = \sum_{i=1}^2 \psi_{2i}(x) w_i + \sum_{i=1}^2 \psi_{3i}(x) w_{,xi}, \quad \phi(x) = \sum_{i=1}^2 \psi_{1i}(x) \phi_i \quad (16-18)$$

The shape functions, ψ_{ij} ($i=1,2,3; j=1,2$), in (15-17) can be expressed as given below.

$$\psi_{11} = 1 - \frac{x}{L_e}, \quad \psi_{12} = \frac{x}{L_e}, \quad (19, 20)$$

$$\psi_{21} = 1 - \frac{3x^2}{L_e^2} + \frac{2x^3}{L_e^3}, \quad \psi_{22} = \frac{3x^2}{L_e^2} - \frac{2x^3}{L_e^3}, \quad \psi_{31} = x - \frac{2x^2}{L_e} + \frac{x^3}{L_e^2}, \quad \psi_{32} = -\frac{x^2}{L_e} + \frac{x^3}{L_e^2} \quad (21-24)$$

Substituting Eqs. (16-18) into Eq. (15) and employing Lagrange's principle, the equations of motions for a porous 2D FG beam can be derived from,

$$\frac{\partial \Pi}{\partial q_i} - \frac{d}{dt} \left(\frac{\partial \Pi}{\partial \dot{q}_i} \right) = 0 \quad (25)$$

where, q_i ($i=1,2,3$) represents generalized nodal displacements u_i , w_i , ϕ_i and β_i . By regarding $\mathbf{m}_e$, $\mathbf{k}_e$, $\mathbf{k}_g$, $\mathbf{u}_e$, and $\mathbf{f}_e$ as matrices respectively representing mass, stiffness, geometric stiffness, displacement, and force for a single element, the equations of motion can be expressed in matrix form as shown below.

$$[\mathbf{m}_e][\ddot{\mathbf{u}}_e] + [\mathbf{k}_e - N_0 \mathbf{k}_g][\mathbf{u}_e] = [\mathbf{f}_e] \quad (26)$$

For a porous 2D functionally graded beam of length L , comprised of N elements such that $L = NL_e$, the ensuing equation of motion can be formulated as follows,

$$[\mathbf{M}][\ddot{\mathbf{U}}] + [\mathbf{K} - N_0 \mathbf{K}_G][\mathbf{U}] = [\mathbf{F}] \quad (27)$$

in which, $\mathbf{M}$, $\mathbf{K}$, $\mathbf{K}_G$, $\mathbf{U}$, and $\mathbf{F}$ are the global mass, stiffness, geometric stiffness, displacement, and force matrices, respectively. Considering $\mathbf{M} = \mathbf{0}$, and $N_0=0$, Eq. (27) can be transformed into the bending problem of a beam as follows.

$$[\mathbf{K}][\mathbf{U}] = [\mathbf{F}] \quad (28)$$

By solving the system of equations provided by Eq. (28), the unknown displacements $\mathbf{U}$ can be determined. When $\mathbf{M} = \mathbf{0}$ and $\mathbf{F} = \mathbf{0}$ are taken into consideration, the buckling problem for a beam can be obtained from Eq. (27) as illustrated below.

$$[\mathbf{K} - N_0 \mathbf{K}_G][\mathbf{U}] = 0 \quad (29)$$

If the solution of Eq. (29) can be sought in the form of $\mathbf{U} = \bar{\mathbf{U}} \exp(N_0 x)$, then Eq. (29) can be transformed into an eigenvalue and eigenvector problem, leading to the determination of critical buckling loads as $N_0 = N_{cr}$.

3. Numerical Results

Following convergence and verification studies, the impact of porosity, material properties, and geometric characteristics on the bending and buckling behaviour of the porous 2D functionally graded beam is investigated through a parametric study. Numerical results have been attained for three distinct boundary conditions, specifically, simply supported (SS), fully clamped at both ends (CC), and cantilever (CF), using a computer code developed in the Matlab programming environment [37]. In the solutions, the beam parameters were selected as $b = 0.1$, $h = 0.1$, $E_0 = 70 \text{ GPa}$, $\rho_0 = 2702 \text{ kg/m}^3$, $\nu = 0.3$, and $q_0 = 10000 \text{ N/m}$. The results presented for the beam deflections (w), critical buckling loads (N_{cr}) and stresses (σ) were provided utilizing the normalization formulas outlined below.

$$\bar{w} = w \frac{100 E_0 b h^3}{q_0 L^4}, \quad \bar{N}_{cr} = N_{cr} \frac{12 L^2}{E_0 b h^3}, \quad \bar{\sigma} = \sigma \frac{b h}{q L} \quad (30-32)$$

3.1. Convergence Analysis

The quantity of elements, denoted as N , required FEM utilization can be ascertained through convergence analysis. Table 1 presents the non-dimensional maximum deflections and critical buckling loads of a beam under different boundary conditions, porosity distributions, and numbers of elements. As observed in the table, the relative percentage differences in maximum deflections and critical buckling loads are below 0.02% between $N=200$ and $N=500$, leading to the selection of $N=200$ as the number of elements.

Table 1. Convergence of the non-dimensional maximum deflection and critical buckling loads according to various numbers of elements ($L/h=5$, $p_x = p_z = 0.4$, $e=0.3$)

N	Type-H			Type-X			Type-O		
	SS	CC	CF	SS	CC	CF	SS	CC	CF
<i>Bending Analysis</i>									
2	14.3615	2.5685	145.0598	15.4777	2.8067	157.7897	13.1383	2.3125	131.2910
4	14.3945	3.5654	146.2552	15.5112	3.7444	158.9355	13.1704	3.3568	132.5219
10	14.4020	3.7436	146.8579	15.5187	3.9124	159.5056	13.1778	3.5429	133.1497
20	14.4031	3.8112	147.0285	15.5198	3.9767	159.6678	13.1788	3.6128	133.3260
50	14.4124	3.8514	147.1031	15.5291	4.0160	159.7402	13.1879	3.6532	133.4016
100	14.4147	3.8569	147.1167	15.5320	4.0215	159.7535	13.1898	3.6587	133.4150
200	14.4153	3.8583	147.1202	15.5325	4.0229	159.7571	13.1905	3.6601	133.4186
500	14.4154	3.8587	147.1213	15.5327	4.0233	159.7580	13.1906	3.6605	133.4198
<i>Buckling Analysis</i>									
2	12.5777	48.6647	2.7645	11.6547	44.5348	2.5399	13.7699	54.0532	3.0565
4	11.1188	40.4373	2.6808	10.3204	38.4486	2.4633	12.1493	43.0174	2.9636
10	10.7245	34.3563	2.6571	9.9568	32.8139	2.4416	11.7152	36.3701	2.9373
20	10.6689	33.5974	2.6537	9.9055	32.0988	2.4385	11.6542	35.5549	2.9335
50	10.6534	33.3896	2.6527	9.8912	31.9025	2.4376	11.6371	35.3322	2.9325
100	10.6512	33.3601	2.6526	9.8891	31.8746	2.4375	11.6347	35.3006	2.9323
200	10.6506	33.3527	2.6525	9.8886	31.8676	2.4374	11.6341	35.2926	2.9323
500	10.6505	33.3507	2.6525	9.8886	31.8657	2.4374	11.6339	35.2905	2.9323

3.2. Verification Studies

A verification study was conducted to compare the mid-span deflections and critical buckling loads between the outcomes of the current study and the findings presented in [21]. Nyugen et al. [21] proposed a two-variable shear beam theory for bending, free vibration and buckling analysis of symmetric porous beams using Ritz's method. With $p_x = p_z = 0$, $\nu = 1/3$, $E_0 = 210 \text{ GPa}$, and $\rho_0 = 7850 \text{ kg/m}^3$ taken into consideration, the materials exhibit similarity in both the present study and the research conducted by Nyugen et al. [21], as well as in the beam theories employed. These similarities enable a meaningful comparison of the study results. However, due to an inconsistency between normalization formulas of the studies, the results obtained in the present study were normalized using the formulas from [21]. The comparison of the non-dimensional mid-span deflections and critical buckling loads for various slenderness (L/h), distribution types, porosity coefficient and boundary conditions are given in Table 2 and 3, respectively. It is evident that the results obtained in the present study align well with the findings from the existing study [31].

3.3. Bending Analysis of porous 2D FG Beams

The maximum deflections and stresses in the bending analysis of porous 2D FG beams were documented for various beam parameters and boundary conditions. Tables 4-6 respectively present the non-dimensional maximum deflections for different slenderness ratios, distribution types, porosity coefficients, and gradation parameters in cases of simply supported, clamped clamped, and

cantilever boundary conditions. The anticipation is that greater deflections will occur with an increase in L/h . However, as the slenderness ratio increases, $\bar{w}$ becomes smaller due to the division by $(L/h)^3$ in the normalization formula (30). As expected, an increase in the gradation parameters leads to a decrease in maximum deflections due to the resulting increase in stiffness. On the other hand, if the porosity coefficient increases, the rigidity of the beam decreases. Therefore, with the increasing of e , the maximum deflections also increase. The integration of porosity distribution functions along the height $\left(-\frac{h}{2}, \frac{h}{2}\right)$ of the beam reveals that the total porosity of the cross-section is nearly the same for Type-H, Type-X, and Type-O. In other words, the variation in maximum deflections is solely attributed to the distribution of porosity along the cross-section. It is evident that the maximum $\bar{w}$ is achieved for Type-X, while the minimum occurs for Type-O. This observation suggests that as the positions of porosities move away from the mid-point of the cross-section, $\bar{w}$ becomes greater. As anticipated, the order of $\bar{w}$ from maximum to minimum for the boundary conditions is obtained as CF, SS and CC. The maximum $\bar{w}$ in the Tables 4-6 is obtained for $L/h=5$, $p_x = p_z = 0$, $e = 0.5$, Type-X and CF.

Table 2. Comparison of the non-dimensional mid-span deflection of beams for $p_x = p_z = 0$, ($\times 10^{-4}$)

L/h	Distribution Type	BCs	Study	e			
				0.2	0.4	0.6	0.8
5	Type-H	SS	Present	0.055	0.065	0.082	0.114
			Nguyen at al. [21]	0.056	0.066	0.083	0.115
		CC	Present	0.015	0.018	0.022	0.031
			Nguyen at al. [21]	0.016	0.018	0.023	0.032
		CF	Present	0.185	0.219	0.274	0.383
			Nguyen at al. [21]	0.186	0.221	0.276	0.386
	Type-X	SS	Present	0.058	0.073	0.099	0.156
			Nguyen at al. [21]	0.058	0.073	0.099	0.156
		CC	Present	0.015	0.019	0.025	0.037
			Nguyen at al. [21]	0.016	0.019	0.025	0.038
		CF	Present	0.194	0.244	0.333	0.526
			Nguyen at al. [21]	0.195	0.246	0.335	0.528
	Type-O	SS	Present	0.052	0.057	0.064	0.074
			Nguyen at al. [21]	0.053	0.058	0.065	0.075
		CC	Present	0.014	0.016	0.019	0.024
			Nguyen at al. [21]	0.015	0.017	0.020	0.026
		CF	Present	0.174	0.192	0.213	0.244
			Nguyen at al. [21]	0.176	0.194	0.216	0.248
20	Type-H	SS	Present	12.852	15.243	19.057	26.623
			Nguyen at al. [21]	12.862	15.255	19.072	26.645
		CC	Present	2.635	3.125	3.907	5.458
			Nguyen at al. [21]	2.644	3.136	3.921	5.477
		CF	Present	43.661	51.785	64.742	90.448
			Nguyen at al. [21]	43.690	51.820	64.786	90.508
	Type-X	SS	Present	13.576	17.276	23.758	38.054
			Nguyen at al. [21]	13.585	17.284	23.763	38.044
		CC	Present	2.777	3.525	4.831	7.705
			Nguyen at al. [21]	2.786	3.534	4.841	7.714
		CF	Present	46.124	58.701	80.734	129.335
			Nguyen at al. [21]	46.149	58.724	80.745	129.292
	Type-O	SS	Present	12.062	13.095	14.328	15.838
			Nguyen at al. [21]	12.073	13.108	14.345	15.863
		CC	Present	2.479	2.701	2.973	3.326
			Nguyen at al. [21]	2.489	2.713	2.989	3.349
		CF	Present	40.975	44.479	48.658	53.763
			Nguyen at al. [21]	41.006	44.516	48.707	53.835

Table 3. Comparison of the non-dimensional critical buckling loads of beams for $p_x = p_z = 0$, ($\times 10^{-4}$)

L/h	Distribution Type	BCs	Study	e			
				0.2	0.4	0.6	0.8
5	Type-H	SS	Present	518.042	436.773	349.358	250.070
			Nguyen at al. [21]	511.954	431.639	345.252	247.131
		CC	Present	1613.928	1360.738	1088.402	779.077
			Nguyen at al. [21]	1556.679	1312.471	1049.795	751.442
		CF	Present	139.457	117.579	94.047	67.319
			Nguyen at al. [21]	139.011	117.203	93.746	67.103
	Type-X	SS	Present	494.375	392.340	288.858	183.251
			Nguyen at al. [21]	489.062	388.620	286.600	182.245
		CC	Present	1568.631	1274.028	966.786	639.077
			Nguyen at al. [21]	1516.516	1235.447	941.392	626.035
		CF	Present	132.250	104.144	75.931	47.565
			Nguyen at al. [21]	131.872	103.892	75.792	47.520
	Type-O	SS	Present	547.083	497.345	444.859	386.023
			Nguyen at al. [21]	540.085	490.227	437.408	377.804
		CC	Present	1671.345	1477.371	1265.200	1016.846
			Nguyen at al. [21]	1608.195	1416.804	1207.306	962.409
		CF	Present	148.298	136.205	123.886	111.014
			Nguyen at al. [21]	147.778	135.665	123.300	110.322
20	Type-H	SS	Present	35.548	29.971	23.973	17.160
			Nguyen at al. [21]	35.518	29.946	23.953	17.146
		CC	Present	139.465	117.586	94.053	67.323
			Nguyen at al. [21]	139.011	117.203	93.746	67.103
		CF	Present	8.931	7.530	6.023	4.311
			Nguyen at al. [21]	8.929	7.528	6.021	4.3101
	Type-X	SS	Present	33.653	26.445	19.231	12.006
			Nguyen at al. [21]	33.629	26.432	19.226	12.009
		CC	Present	132.258	104.151	75.936	47.568
			Nguyen at al. [21]	131.872	103.892	75.792	47.520
		CF	Present	8.451	6.637	4.823	3.009
			Nguyen at al. [21]	8.450	6.637	4.824	3.010
	Type-O	SS	Present	37.874	34.886	31.882	28.840
			Nguyen at al. [21]	37.839	34.849	31.842	28.793
		CC	Present	148.307	136.214	123.894	111.021
			Nguyen at al. [21]	147.778	135.665	123.300	110.322
		CF	Present	9.520	8.775	8.029	7.281
			Nguyen at al. [21]	9.518	8.773	8.027	7.278

Table 4. The non-dimensional maximum deflections of SS beams with various slenderness ratios, porosity coefficients, and gradation parameters

Dist. Type	p_z	e	$L/h=5$				$L/h=20$			
			p_x				p_x			
			0	0.25	0.5	1	0	0.25	0.5	1
Type-H	0	0	17.1835	15.1874	13.4646	10.6803	15.7225	13.8916	12.3043	9.7230
		0.1	18.3637	16.2305	14.3894	11.4139	16.8024	14.8457	13.1494	10.3908
		0.3	21.4032	18.9170	16.7712	13.3031	19.5835	17.3029	15.3259	12.1107
		0.5	25.9629	22.9471	20.3441	16.1372	23.7556	20.9891	18.5910	14.6907
	0.25	0	15.1693	13.4072	11.8864	9.4284	13.8821	12.2654	10.8640	8.5849
		0.1	16.2113	14.3281	12.7028	10.0760	14.8356	13.1079	11.6102	9.1745
		0.3	18.8946	16.6998	14.8054	11.7437	17.2912	15.2776	13.5320	10.6931
		0.5	22.9198	20.2574	17.9595	14.2456	20.9748	18.5322	16.4148	12.9711
	0.5	0	13.4000	11.8434	10.4999	8.3284	12.2693	10.8405	9.6019	7.5875
		0.1	14.3204	12.6569	11.2210	8.9004	13.1120	11.5851	10.2614	8.1086
		0.3	16.6907	14.7519	13.0784	10.3736	15.2823	13.5026	11.9599	9.4508
		0.5	20.2464	17.8945	15.8645	12.5836	18.5380	16.3792	14.5078	11.4642
	1	0	10.4746	9.2577	8.2073	6.5094	9.6104	8.4912	7.5210	5.9431
		0.1	11.1940	9.8936	8.7711	6.9565	10.2705	9.0744	8.0376	6.3513
		0.3	13.0469	11.5312	10.2228	8.1079	11.9704	10.5764	9.3680	7.4026
		0.5	15.8263	13.9878	12.4007	9.8352	14.5206	12.8296	11.3637	8.9796
Type-X	0	0	17.1835	15.1874	13.4646	10.6803	15.7225	13.8916	12.3043	9.7230
		0.1	18.7760	16.5947	14.7117	11.6676	17.2423	15.2344	13.4937	10.6627
		0.3	23.0825	20.4003	18.0838	14.3364	21.3770	18.8876	16.7293	13.2192
		0.5	30.0545	26.5610	23.5423	18.6551	28.1274	24.8517	22.0118	17.3927
	0.25	0	15.1693	13.4072	11.8864	9.4284	13.8821	12.2654	10.8640	8.5849
		0.1	16.5738	14.6484	12.9862	10.2990	15.2224	13.4497	11.9129	9.4136
		0.3	20.3708	18.0037	15.9593	12.6521	18.8680	16.6707	14.7658	11.6676
		0.5	26.5162	23.4340	20.7707	16.4588	24.8179	21.9276	19.4219	15.3462
	0.5	0	13.4000	11.8434	10.4999	8.3284	12.2693	10.8405	9.6019	7.5875
		0.1	14.6368	12.9364	11.4684	9.0951	13.4497	11.8834	10.5256	8.3173
		0.3	17.9786	15.8894	14.0851	11.1661	16.6581	14.7182	13.0364	10.3011
		0.5	23.3824	20.6644	18.3158	14.5133	21.8898	19.3405	17.1304	13.5356
	1	0	10.4746	9.2577	8.2073	6.5094	9.6104	8.4912	7.5210	5.9431
		0.1	11.4296	10.1017	8.9552	7.1014	10.5220	9.2967	8.2344	6.5068
		0.3	14.0040	12.3766	10.9710	8.6968	12.9936	11.4804	10.1686	8.0350
		0.5	18.1525	16.0424	14.2189	11.2666	17.0093	15.0284	13.3110	10.5177
Type-O	0	0	17.1835	15.1874	13.4646	10.6803	15.7225	13.8916	12.3043	9.7230
		0.1	17.9022	15.8229	14.0286	11.1296	16.3173	14.4171	12.7699	10.0910
		0.3	19.5680	17.2959	15.3362	12.1726	17.6551	15.5991	13.8170	10.9188
		0.5	21.6613	19.1473	16.9806	13.4871	19.2375	16.9974	15.0557	11.8983
	0.25	0	15.1693	13.4072	11.8864	9.4284	13.8821	12.2654	10.8640	8.5849
		0.1	15.8051	13.9694	12.3852	9.8257	14.4086	12.7307	11.2762	8.9107
		0.3	17.2793	15.2729	13.5424	10.7487	15.5939	13.7780	12.2039	9.6441
		0.5	19.1331	16.9125	14.9986	11.9128	16.9978	15.0185	13.3029	10.5131
	0.5	0	13.4000	11.8434	10.4999	8.3284	12.2693	10.8405	9.6019	7.5875
		0.1	13.9649	12.3429	10.9431	8.6814	12.7384	11.2550	9.9690	7.8777
		0.3	15.2766	13.5028	11.9727	9.5025	13.7966	12.1900	10.7973	8.5325
		0.5	16.9298	14.9649	13.2713	10.5404	15.0553	13.3021	11.7826	9.3116
	1	0	10.4746	9.2577	8.2073	6.5094	9.6104	8.4912	7.5210	5.9431
		0.1	10.9263	9.6571	8.5617	6.7915	9.9891	8.8258	7.8174	6.1774
		0.3	11.9808	10.5896	9.3893	7.4512	10.8507	9.5871	8.4918	6.7105
		0.5	13.3213	11.7750	10.4420	8.2919	11.8916	10.5068	9.3065	7.3547

Table 5. The non-dimensional maximum deflections of CC beams with various slenderness ratios, porosity coefficients, and gradation parameters

Dist. Type	p_z	e	$L/h=5$				$L/h=20$			
			p_x				p_x			
			0	0.25	0.5	1	0	0.25	0.5	1
Type-H	0	0	4.6150	4.0753	3.6042	2.8304	3.2213	2.8434	2.5111	1.9610
		0.1	4.9319	4.3553	3.8517	3.0248	3.4425	3.0387	2.6835	2.0957
		0.3	5.7483	5.0762	4.4893	3.5254	4.0123	3.5417	3.1277	2.4426
		0.5	6.9729	6.1576	5.4457	4.2765	4.8671	4.2962	3.7940	2.9629
	0.25	0	4.0720	3.5959	3.1801	2.4973	2.8440	2.5105	2.2170	1.7314
		0.1	4.3517	3.8429	3.3986	2.6689	3.0394	2.6829	2.3693	1.8503
		0.3	5.0720	4.4790	3.9611	3.1106	3.5425	3.1270	2.7615	2.1566
		0.5	6.1525	5.4331	4.8050	3.7733	4.2972	3.7931	3.3498	2.6160
	0.5	0	3.5916	3.1717	2.8050	2.2026	2.5133	2.2185	1.9592	1.5300
		0.1	3.8383	3.3895	2.9976	2.3539	2.6859	2.3708	2.0937	1.6351
		0.3	4.4736	3.9506	3.4938	2.7436	3.1305	2.7633	2.4403	1.9057
		0.5	5.4267	4.7922	4.2381	3.3280	3.7974	3.3520	2.9601	2.3117
	1	0	2.7910	2.4647	2.1796	1.7114	1.9675	1.7367	1.5337	1.1977
		0.1	2.9827	2.6340	2.3293	1.8289	2.1026	1.8560	1.6390	1.2800
		0.3	3.4764	3.0699	2.7149	2.1317	2.4506	2.1632	1.9103	1.4918
		0.5	4.2170	3.7239	3.2933	2.5858	2.9727	2.6240	2.3173	1.8096
Type-X	0	0	4.6150	4.0753	3.6042	2.8304	3.2213	2.8434	2.5111	1.9610
		0.1	4.9934	4.4095	3.8995	3.0618	3.5291	3.1151	2.7510	2.1483
		0.3	5.9967	5.2953	4.6824	3.6750	4.3651	3.8531	3.4026	2.6570
		0.5	7.5756	6.6894	5.9141	4.6392	5.7269	5.0551	4.4640	3.4856
	0.25	0	4.0720	3.5959	3.1801	2.4973	2.8440	2.5105	2.2170	1.7314
		0.1	4.4057	3.8905	3.4405	2.7014	3.1155	2.7501	2.4286	1.8966
		0.3	5.2903	4.6716	4.1308	3.2420	3.8526	3.4007	3.0031	2.3451
		0.5	6.6821	5.9003	5.2165	4.0920	5.0529	4.4602	3.9387	3.0754
	0.5	0	3.5916	3.1717	2.8050	2.2026	2.5133	2.2185	1.9592	1.5300
		0.1	3.8854	3.4311	3.0342	2.3823	2.7523	2.4295	2.1455	1.6755
		0.3	4.6640	4.1185	3.6417	2.8581	3.4011	3.0021	2.6511	2.0702
		0.5	5.8879	5.1991	4.5966	3.6056	4.4565	3.9337	3.4737	2.7124
	1	0	2.7910	2.4647	2.1796	1.7114	1.9675	1.7367	1.5337	1.1977
		0.1	3.0178	2.6649	2.3566	1.8500	2.1521	1.8996	1.6776	1.3100
		0.3	3.6175	3.1944	2.8245	2.2165	2.6518	2.3408	2.0671	1.6141
		0.5	4.5576	4.0244	3.5579	2.7907	3.4620	3.0559	2.6985	2.1071
Type-O	0	0	4.6150	4.0753	3.6042	2.8304	3.2213	2.8434	2.5111	1.9610
		0.1	4.8573	4.2894	3.7936	2.9796	3.3467	2.9542	2.6089	2.0375
		0.3	5.4506	4.8136	4.2576	3.3454	3.6315	3.2055	2.8309	2.2110
		0.5	6.2725	5.5397	4.9005	3.8526	3.9747	3.5085	3.0986	2.4204
	0.25	0	4.0720	3.5959	3.1801	2.4973	2.8440	2.5105	2.2170	1.7314
		0.1	4.2859	3.7849	3.3474	2.6291	2.9551	2.6085	2.3036	1.7990
		0.3	4.8099	4.2478	3.7571	2.9521	3.2073	2.8311	2.5002	1.9528
		0.5	5.5357	4.8891	4.3249	3.4001	3.5116	3.0998	2.7376	2.1384
	0.5	0	3.5916	3.1717	2.8050	2.2026	2.5133	2.2185	1.9592	1.5300
		0.1	3.7808	3.3387	2.9528	2.3191	2.6121	2.3057	2.0362	1.5902
		0.3	4.2441	3.7481	3.3151	2.6047	2.8370	2.5043	2.2116	1.7273
		0.5	4.8860	4.3152	3.8173	3.0009	3.1094	2.7447	2.4240	1.8934
	1	0	2.7910	2.4647	2.1796	1.7114	1.9675	1.7367	1.5337	1.1977
		0.1	2.9393	2.5956	2.2955	1.8027	2.0470	1.8069	1.5957	1.2462
		0.3	3.3030	2.9169	2.5799	2.0268	2.2295	1.9680	1.7380	1.3574
		0.5	3.8073	3.3624	2.9743	2.3378	2.4533	2.1656	1.9125	1.4939

Table 6. The non-dimensional maximum deflections of CF beams with various slenderness ratios, porosity coefficients, and gradation parameters

Dist. Type	p_z	e	$L/h=5$				$L/h=20$			
			p_x				p_x			
			0	0.25	0.5	1	0	0.25	0.5	1
Type-H	0	0	156.1015	148.4143	141.3459	128.8193	150.3875	143.1587	136.4959	124.6505
		0.1	166.8234	158.6082	151.0543	137.6674	160.7170	152.9916	145.8712	133.2121
		0.3	194.4361	184.8612	176.0569	160.4542	187.3190	178.3149	170.0159	155.2615
		0.5	235.8582	224.2434	213.5635	194.6368	227.2248	216.3025	206.2356	188.3379
	0.25	0	137.8193	131.0327	124.7924	113.7334	132.7848	126.4021	120.5192	110.0603
		0.1	147.2855	140.0328	133.3639	121.5452	141.9052	135.0841	128.7971	117.6199
		0.3	171.6643	163.2111	155.4383	141.6635	165.3934	157.4432	150.1157	137.0883
		0.5	208.2351	197.9811	188.5524	171.8430	200.6283	190.9845	182.0959	166.2932
	0.5	0	121.7826	115.7866	110.2731	100.5021	117.3608	111.7196	106.5201	97.2761
		0.1	130.1473	123.7395	117.8473	107.4051	125.4219	119.3931	113.8365	103.9576
		0.3	151.6893	144.2209	137.3535	125.1828	146.1817	139.1551	132.6788	121.1647
		0.5	184.0047	174.9453	166.6148	151.8514	177.3239	168.8003	160.9443	146.9773
	1	0	95.3136	90.6235	86.3106	78.6666	91.9352	87.5162	83.4433	76.2022
		0.1	101.8602	96.8480	92.2389	84.0699	98.2498	93.5274	89.1747	81.4362
		0.3	118.7202	112.8784	107.5063	97.9852	114.5122	109.0080	103.9350	94.9156
		0.5	144.0120	136.9256	130.4092	118.8596	138.9075	132.2308	126.0769	115.1361
Type-X	0	0	156.1015	148.4143	141.3459	128.8193	150.3875	143.1587	136.4959	124.6505
		0.1	170.9512	162.5418	154.8084	141.1016	164.9506	157.0222	149.7148	136.7230
		0.3	211.2586	200.8921	191.3568	174.4508	204.5797	194.7482	185.6865	169.5754
		0.5	276.8563	263.3127	250.8513	228.7483	269.2997	256.3606	244.4343	223.2297
	0.25	0	137.8193	131.0327	124.7924	113.7334	132.7848	126.4021	120.5192	110.0603
		0.1	150.9150	143.4915	136.6648	124.5649	145.6277	138.6282	132.1767	120.7068
		0.3	186.4538	177.3048	168.8894	153.9687	180.5685	171.8910	163.8928	149.6727
		0.5	244.2736	232.3241	221.3295	201.8281	237.6144	226.1976	215.6746	196.9649
	0.5	0	121.7826	115.7866	110.2731	100.5021	117.3608	111.7196	106.5201	97.2761
		0.1	133.3154	126.7585	120.7286	110.0410	128.6713	122.4868	116.7866	106.6523
		0.3	164.5929	156.5174	149.0893	135.9190	159.4225	151.7612	144.6997	132.1449
		0.5	215.4338	204.8958	195.1999	178.0017	209.5821	199.5123	190.2307	173.7283
	1	0	95.3136	90.6235	86.3106	78.6666	91.9352	87.5162	83.4433	76.2022
		0.1	104.2202	99.0969	94.3853	86.0334	100.6706	95.8321	91.3725	83.4438
		0.3	128.3145	122.0214	116.2326	105.9683	124.3596	118.3834	112.8752	103.0818
		0.5	167.3409	159.1575	151.6277	138.2714	162.8604	155.0356	147.8232	134.9998
Type-O	0	0	156.1015	148.4143	141.3459	128.8193	150.3875	143.1587	136.4959	124.6505
		0.1	162.2470	154.2481	146.8939	133.8628	156.0510	148.5493	141.6352	129.3429
		0.3	176.2413	167.5264	159.5161	145.3280	168.7702	160.6554	153.1764	139.8802
		0.5	193.2226	183.6233	174.8041	159.1926	183.7696	174.9308	166.7848	152.3034
	0.25	0	137.8193	131.0327	124.7924	113.7334	132.7848	126.4021	120.5192	110.0603
		0.1	143.2580	136.1956	129.7025	118.1970	137.7988	131.1746	125.0692	114.2147
		0.3	155.6504	147.9543	140.8803	128.3505	149.0683	141.9008	135.2949	123.5510
		0.5	170.7047	162.2249	154.4342	140.6430	162.3772	154.5674	147.3697	134.5741
	0.5	0	121.7826	115.7866	110.2731	100.5021	117.3608	111.7196	106.5201	97.2761
		0.1	126.6224	120.3812	114.6429	104.4747	121.8281	115.9717	110.5739	100.9775
		0.3	137.6710	130.8652	124.6096	113.5289	131.8914	125.5500	119.7053	109.3147
		0.5	151.1366	143.6310	136.7350	124.5274	143.8265	136.9091	130.5337	119.2002
	1	0	95.3136	90.6235	86.3106	78.6666	91.9352	87.5162	83.4433	76.2022
		0.1	99.2057	94.3190	89.8257	81.8631	95.5437	90.9509	86.7179	79.1922
		0.3	108.1542	102.8119	97.9010	89.2014	103.7427	98.7549	94.1579	85.9852
		0.5	119.1955	113.2825	107.8493	98.2297	113.6219	108.1576	103.1214	94.1685

The normal and shear stresses for various gradation parameters along the simple supported 2D FG beam was given in Fig. 5 and 6, respectively. A variation in p_x does not lead to any significant change in normal stresses. On the other hand, an increase in p_z results in higher absolute normal stresses at the top of the beam and lower stresses at the bottom. This indicates that changes in rigidity along the height predominantly impact normal stresses. As anticipated, when both p_x and p_z increase, the normal stresses exhibit a variation similar to that caused by an increase in p_z alone. An increase in p_x causes the shear stress distribution to shift to the center, whereas an increase in p_z leads to an upward shift in the shear stress distribution. Notably, the change in stress distributions is more pronounced for variations in p_z . If both p_x and p_z increase, a combined effect is observed in shear stresses. Fig. 7 shows the distributions of normal and shear stresses along the beam for the cases of non-porous, Type-H, Type-X and Type-O. Slight differences are observed between the non-porous beam and the porous beams with different porosity distributions. The maximum stresses in the beams exhibit minimal changes due to porosity. The stress distributions indicate only slight shifts resulting from different porosity distributions, with the changes primarily concentrated around the supports.

3.3. Buckling Analysis of porous 2D FG Beams

The findings pertaining to critical buckling loads and mode shapes for porous 2D FG beams were elaborated upon. The non-dimensional critical buckling loads for various L/h , distribution types, e , p_x , and p_z in cases of SS, CC and CF in Tables 5-7. Contrary to the expected trend, similar to the non-dimensional deflections, the critical buckling loads increase as the slenderness ratio increases due to the normalization process. An increase in gradation parameters results in an increase in the stiffness of the beam, leading to smaller $\bar{N}_{cr}$ values. If the porosity coefficient increases, $\bar{N}_{cr}$ becomes smaller due to a loss in the rigidity of the beam. The buckling of the beam occurs in the direction of the minimum moment of inertia of the cross-section. If the porosity moves to the edges of the cross-section, the moment of inertia decreases. Therefore, as anticipated, the smallest $\bar{N}_{cr}$ was obtained for Type-X, while the greatest $\bar{N}_{cr}$ was obtained for Type-O. Consistent with findings in the literature, maximum critical loads were obtained for CC, whereas minimum values were observed for CF. The parameters for the maximum $\bar{N}_{cr}$ in Tables 5-7 are $L/h=20$, $p_x=p_z=1$, $e=0$, Type-O and CC.

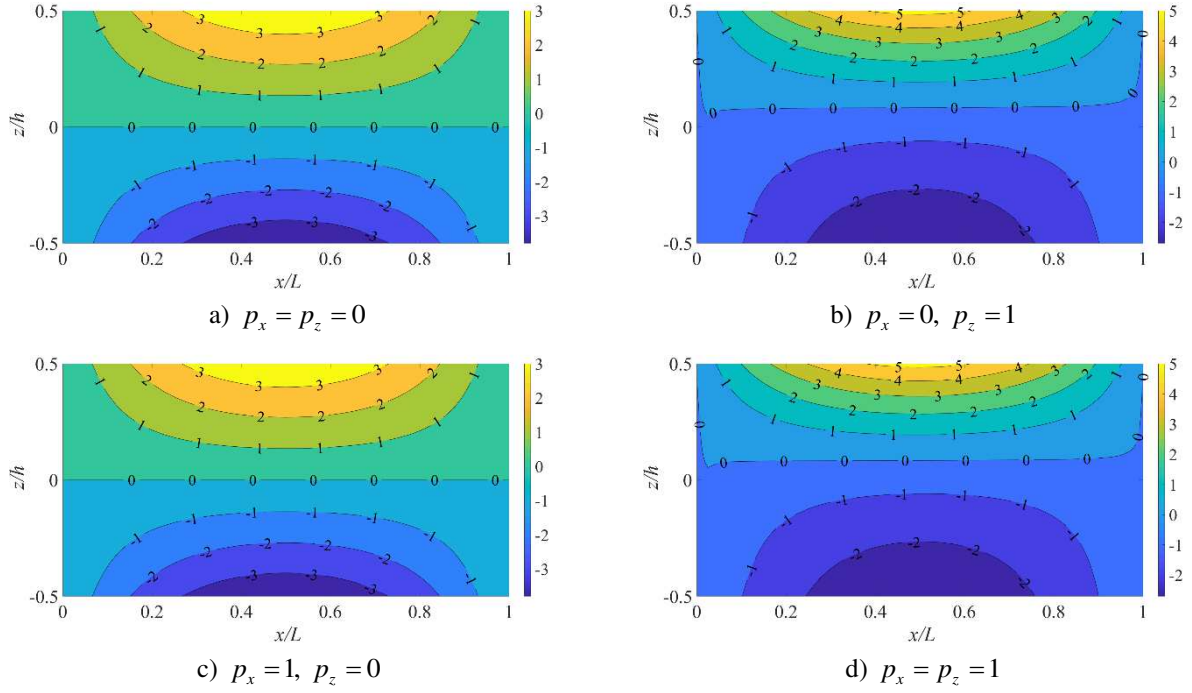


Fig. 5. The distribution of the $\bar{\sigma}_{xx}$ along the width and height of the non-porous SS 2D FG beams for various p_x and p_z ($L/h=5$)

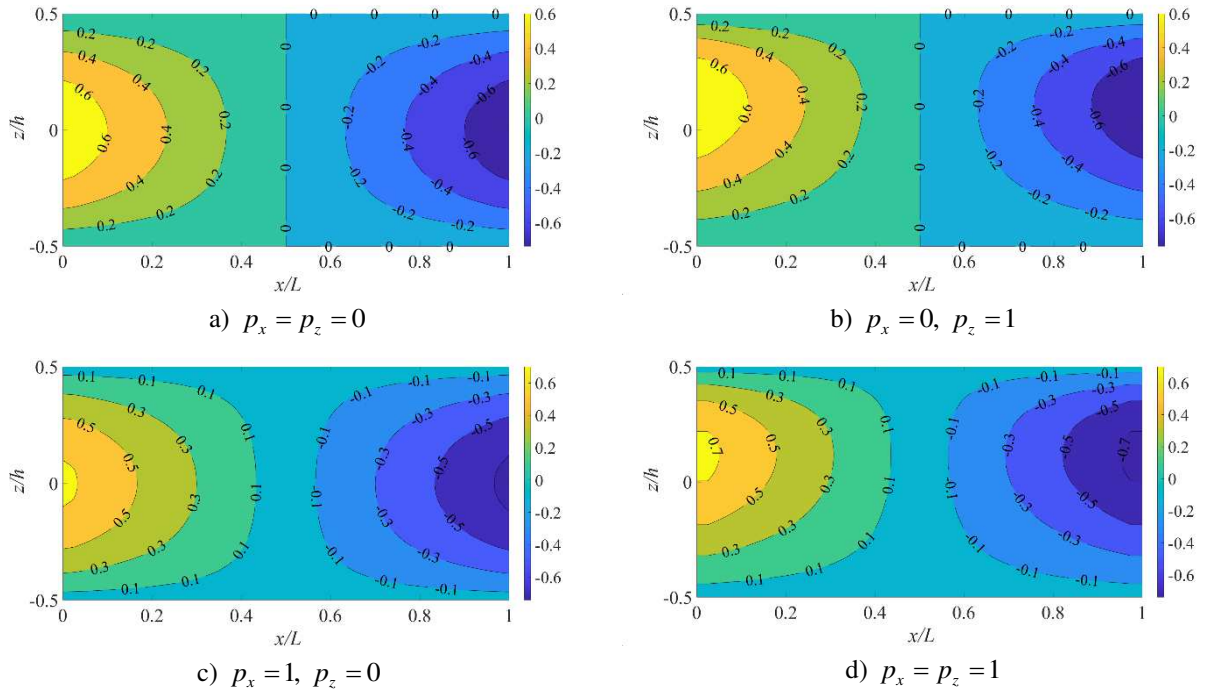
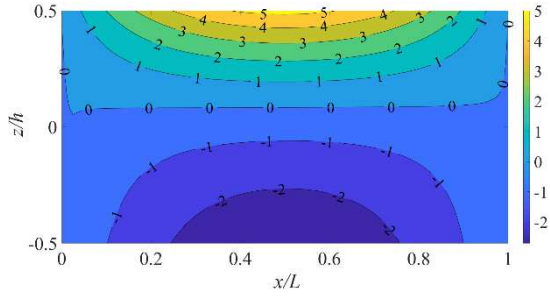
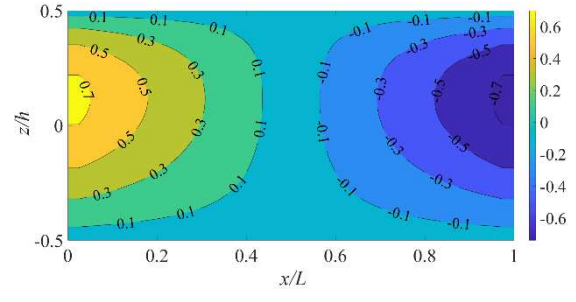


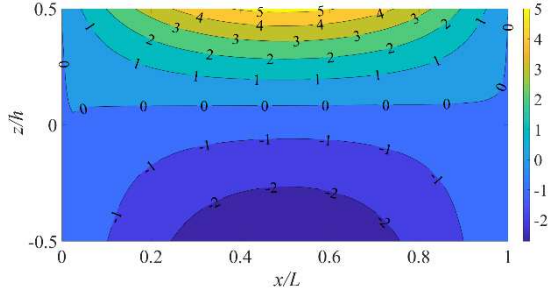
Fig. 6. The distribution of the $\bar{\tau}_{xz}$ along the width and height of the non-porous SS 2D FG beams for various p_x and p_z ($L/h=5$)



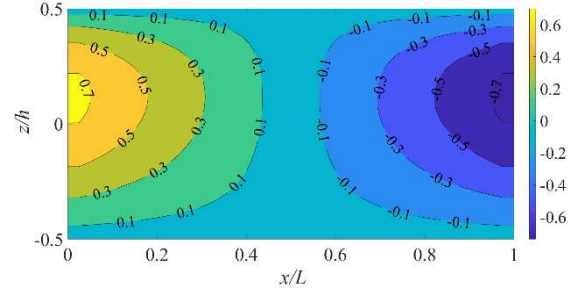
a) σ_{xx} , $e=0$



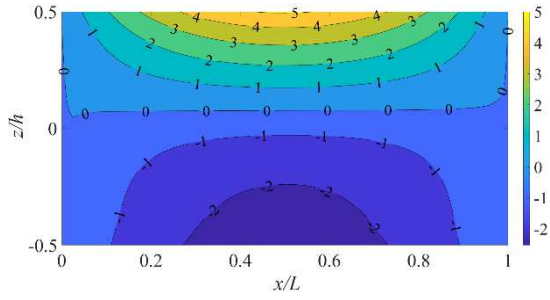
e) τ_{xz} , $e=0$



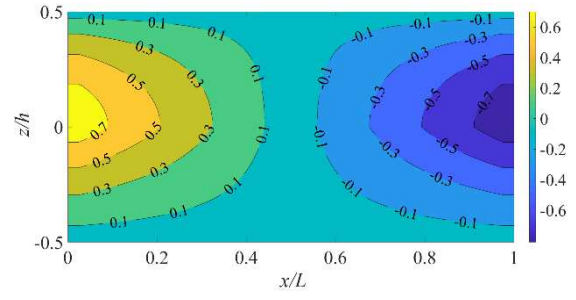
b) σ_{xx} , Type-H, $e=0.3$



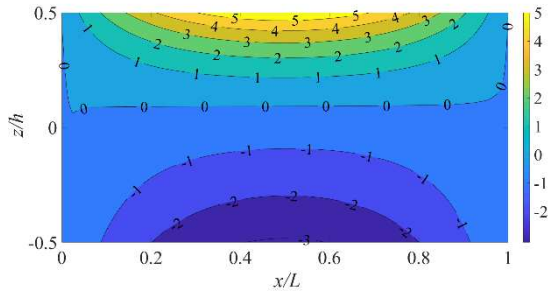
f) τ_{xz} , Type-H, $e=0.3$



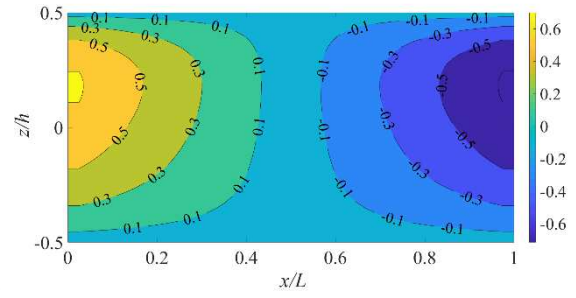
c) σ_{xx} , Type-X, $e=0.3$



g) τ_{xz} , Type-X, $e=0.3$



d) σ_{xx} , Type-O, $e=0.3$



h) τ_{xz} , Type-O, $e=0.3$

Fig. 7. The distribution of the $\bar{\sigma}_{xx}$ and $\bar{\tau}_{xz}$ along the width and height of the SS 2D FG beams for various porosity types ($p_x = p_z = 1$, $L/h = 5$)

Table 7. The non-dimensional critical buckling loads of SS beams with various slenderness ratios, porosity coefficients, and gradation parameters

Dist. Type	p_z	e	$L/h=5$				$L/h=20$			
			p_x				p_x			
			0	0.25	0.5	1	0	0.25	0.5	1
Type-H	0	0	8.9520	10.1208	11.3902	14.2322	9.8069	11.0934	12.5052	15.7281
		0.1	8.3767	9.4703	10.6581	13.3175	9.1765	10.3804	11.7015	14.7171
		0.3	7.1871	8.1254	9.1445	11.4262	7.8733	8.9062	10.0397	12.6271
		0.5	5.9248	6.6984	7.5385	9.4195	6.4906	7.3421	8.2765	10.4095
	0.25	0	10.1407	11.4647	12.9027	16.1223	11.1070	12.5640	14.1631	17.8132
		0.1	9.4889	10.7278	12.0734	15.0861	10.3931	11.7565	13.2529	16.6683
		0.3	8.1414	9.2043	10.3588	12.9437	8.9172	10.0870	11.3708	14.3012
		0.5	6.7115	7.5878	8.5396	10.6705	7.3511	8.3155	9.3738	11.7896
	0.5	0	11.4798	12.9788	14.6068	18.2524	12.5670	14.2156	16.0249	20.1548
		0.1	10.7420	12.1446	13.6680	17.0793	11.7593	13.3020	14.9950	18.8595
		0.3	9.2165	10.4199	11.7270	14.6538	10.0893	11.4129	12.8655	16.1811
		0.5	7.5979	8.5899	9.6674	12.0803	8.3174	9.4085	10.6060	13.3394
	1	0	14.6868	16.6047	18.6884	23.3566	16.0440	18.1488	20.4587	25.7316
		0.1	13.7429	15.5375	17.4873	21.8555	15.0129	16.9823	19.1438	24.0778
		0.3	11.7912	13.3310	15.0039	18.7517	12.8808	14.5706	16.4251	20.6584
		0.5	9.7204	10.9898	12.3688	15.4585	10.6187	12.0117	13.5405	17.0303
Type-X	0	0	8.9520	10.1208	11.3902	14.2322	9.8069	11.0934	12.5052	15.7281
		0.1	8.1935	9.2635	10.4262	13.0314	8.9425	10.1156	11.4031	14.3422
		0.3	6.6664	7.5374	8.4848	10.6122	7.2130	8.1592	9.1979	11.5690
		0.5	5.1214	5.7910	6.5202	8.1618	5.4820	6.2013	6.9907	8.7934
	0.25	0	10.1407	11.4647	12.9027	16.1223	11.1070	12.5640	14.1631	17.8132
		0.1	9.2823	10.4945	11.8116	14.7633	10.1291	11.4579	12.9163	16.2453
		0.3	7.5538	8.5408	9.6144	12.0250	8.1722	9.2443	10.4210	13.1075
		0.5	5.8048	6.5637	7.3903	9.2510	6.2131	7.0282	7.9229	9.9660
	0.5	0	11.4798	12.9788	14.6068	18.2524	12.5670	14.2156	16.0249	20.1548
		0.1	10.5108	11.8835	13.3751	16.7181	11.4642	12.9681	14.6187	18.3866
		0.3	8.5590	9.6774	10.8939	13.6258	9.2563	10.4706	11.8034	14.8464
		0.5	6.5829	7.4435	8.3809	10.4912	7.0442	7.9683	8.9828	11.2991
	1	0	14.6868	16.6047	18.6884	23.3566	16.0440	18.1488	20.4587	25.7316
		0.1	13.4608	15.2190	17.1299	21.4145	14.6541	16.5765	18.6864	23.5029
		0.3	10.9887	12.4246	13.9868	17.4963	11.8668	13.4236	15.1324	19.0336
		0.5	8.4797	9.5884	10.7961	13.5155	9.0654	10.2547	11.5602	14.5413
Type-O	0	0	8.9520	10.1208	11.3902	14.2322	9.8069	11.0934	12.5052	15.7281
		0.1	8.5917	9.7133	10.9307	13.6538	9.4493	10.6889	12.0492	15.1542
		0.3	7.8582	8.8834	9.9947	12.4744	8.7331	9.8787	11.1358	14.0046
		0.5	7.0959	8.0207	9.0212	11.2450	8.0145	9.0658	10.2191	12.8507
	0.25	0	10.1407	11.4647	12.9027	16.1223	11.1070	12.5640	14.1631	17.8132
		0.1	9.7318	11.0021	12.3812	15.4659	10.7010	12.1048	13.6453	17.1616
		0.3	8.8991	10.0601	11.3187	14.1272	9.8875	11.1845	12.6077	15.8558
		0.5	8.0336	9.0806	10.2134	12.7315	9.0705	10.2603	11.5657	14.5440
	0.5	0	11.4798	12.9788	14.6068	18.2524	12.5670	14.2156	16.0249	20.1548
		0.1	11.0144	12.4522	14.0131	17.5053	12.1042	13.6920	15.4346	19.4120
		0.3	10.0659	11.3792	12.8030	15.9807	11.1755	12.6415	14.2502	17.9214
		0.5	9.0793	10.2627	11.5432	14.3903	10.2409	11.5842	13.0580	16.4207
	1	0	14.6868	16.6047	18.6884	23.3566	16.0440	18.1488	20.4587	25.7316
		0.1	14.0783	15.9164	17.9124	22.3803	15.4356	17.4605	19.6828	24.7552
		0.3	12.8359	14.5108	16.3275	20.3847	14.2097	16.0737	18.1192	22.7875
		0.5	11.5399	13.0444	14.6731	18.2979	12.9655	14.6663	16.5323	20.7902

Table 8. The non-dimensional critical buckling loads of CC beams with various slenderness ratios, porosity coefficients, and gradation parameters

Dist. Type	p_z	e	$L/h=5$				$L/h=20$			
			p_x				p_x			
			0	0.25	0.5	1	0	0.25	0.5	1
Type-H	0	0	28.0291	31.6774	35.6121	44.2986	38.4936	43.5625	49.1715	62.1675
		0.1	26.2276	29.6415	33.3232	41.4514	36.0195	40.7626	46.0111	58.1719
		0.3	22.5029	25.4320	28.5909	35.5647	30.9042	34.9737	39.4769	49.9107
		0.5	18.5509	20.9655	23.5697	29.3188	25.4767	28.8316	32.5439	41.1452
	0.25	0	31.7639	35.8985	40.3578	50.2038	43.5986	49.3397	55.6926	70.4124
		0.1	29.7224	33.5913	37.7640	46.9771	40.7964	46.1686	52.1132	65.8869
		0.3	25.5014	28.8209	32.4010	40.3057	35.0028	39.6120	44.7124	56.5300
		0.5	21.0228	23.7593	26.7106	33.2271	28.8555	32.6552	36.8599	46.6021
	0.5	0	36.0031	40.6898	45.7456	56.9123	49.3353	55.8320	63.0209	79.6781
		0.1	33.6891	38.0747	42.8055	53.2545	46.1645	52.2436	58.9705	74.5571
		0.3	28.9048	32.6675	36.7265	45.6916	39.6085	44.8242	50.5959	63.9689
		0.5	23.8285	26.9304	30.2765	37.6671	32.6523	36.9521	41.7101	52.7345
	1	0	46.2845	52.3115	58.8175	73.2067	63.0140	71.3120	80.4948	101.7733
		0.1	43.3097	48.9493	55.0372	68.5016	58.9640	66.7287	75.3213	95.2322
		0.3	37.1591	41.9978	47.2212	58.7734	50.5903	57.2523	64.6246	81.7079
		0.5	30.6331	34.6221	38.9280	48.4515	41.7055	47.1975	53.2751	67.3581
Type-X	0	0	28.0291	31.6774	35.6121	44.2986	38.4936	43.5625	49.1715	62.1675
		0.1	25.8735	29.2430	32.8812	40.9315	35.1290	39.7550	44.8743	56.7373
		0.3	21.4802	24.2810	27.3134	34.0580	28.3874	32.1260	36.2639	45.8557
		0.5	16.9349	19.1463	21.5486	26.9258	21.6245	24.4726	27.6257	34.9375
	0.25	0	31.7639	35.8985	40.3578	50.2038	43.5986	49.3397	55.6926	70.4124
		0.1	29.3225	33.1413	37.2649	46.3900	39.7918	45.0319	50.8307	64.2685
		0.3	24.3468	27.5214	30.9587	38.6045	32.1633	36.3991	41.0874	51.9551
		0.5	19.1985	21.7056	24.4290	30.5255	24.5086	27.7366	31.3102	39.5972
	0.5	0	36.0031	40.6898	45.7456	56.9123	49.3353	55.8320	63.0209	79.6781
		0.1	33.2408	37.5702	42.2458	52.5959	45.0413	50.9727	57.5366	72.7476
		0.3	27.6110	31.2114	35.1101	43.7845	36.4329	41.2311	46.5418	58.8525
		0.5	21.7847	24.6296	27.7203	34.6398	27.7884	31.4484	35.5003	44.8963
	1	0	46.2845	52.3115	58.8175	73.2067	63.0140	71.3120	80.4948	101.7733
		0.1	42.7585	48.3290	54.3488	67.6903	57.5974	65.1824	73.5767	93.0305
		0.3	35.5712	40.2106	45.2367	56.4295	46.7225	52.8758	59.6867	75.4757
		0.5	28.1276	31.8013	35.7936	44.7369	35.7690	40.4802	45.6959	57.7912
Type-O	0	0	28.0291	31.6774	35.6121	44.2986	38.4936	43.5625	49.1715	62.1675
		0.1	26.6638	30.1325	33.8687	42.0975	37.0586	41.9384	47.3377	59.8460
		0.3	23.8348	26.9309	30.2547	37.5281	34.1725	38.6718	43.6491	55.1753
		0.5	20.8021	23.4979	26.3769	32.6129	31.2495	35.3636	39.9127	50.4414
	0.25	0	31.7639	35.8985	40.3578	50.2038	43.5986	49.3397	55.6926	70.4124
		0.1	30.2154	34.1463	38.3806	47.7074	41.9694	47.4958	53.6107	67.7767
		0.3	27.0068	30.5151	34.2817	42.5254	38.6915	43.7858	49.4214	62.4720
		0.5	23.5676	26.6220	29.8843	36.9521	35.3696	40.0259	45.1751	57.0923
	0.5	0	36.0031	40.6898	45.7456	56.9123	49.3353	55.8320	63.0209	79.6781
		0.1	34.2435	38.6987	43.4989	54.0762	47.4786	53.7305	60.6482	76.6744
		0.3	30.5977	34.5729	38.8419	48.1897	43.7391	49.4980	55.8689	70.6228
		0.5	26.6913	30.1511	33.8477	41.8617	39.9424	45.2006	51.0157	64.4746
	1	0	46.2845	52.3115	58.8175	73.2067	63.0140	71.3120	80.4948	101.7733
		0.1	43.9999	49.7265	55.9011	69.5273	60.5767	68.5535	77.3801	97.8307
		0.3	39.2677	44.3715	49.8578	61.8939	55.6493	62.9767	71.0830	89.8582
		0.5	34.2039	38.6402	43.3863	53.7029	50.6130	57.2762	64.6457	81.7047

Table 9. The non-dimensional critical buckling loads of CF beams with various slenderness ratios, porosity coefficients, and gradation parameters

Dist. Type	p_z	e	$L/h=5$				$L/h=20$			
			p_x				p_x			
			0	0.25	0.5	1	0	0.25	0.5	1
Type-H	0	0	2.4057	2.5933	2.7860	3.1851	2.4635	2.6497	2.8410	3.2376
		0.1	2.2511	2.4266	2.6069	2.9804	2.3051	2.4793	2.6584	3.0295
		0.3	1.9313	2.0820	2.2367	2.5571	1.9778	2.1273	2.2809	2.5993
		0.5	1.5922	1.7164	1.8439	2.1080	1.6304	1.7537	1.8803	2.1428
	0.25	0	2.7247	2.9372	3.1555	3.6074	2.7900	3.0009	3.2177	3.6668
		0.1	2.5496	2.7484	2.9527	3.3756	2.6107	2.8081	3.0109	3.4312
		0.3	2.1875	2.3581	2.5333	2.8962	2.2399	2.4093	2.5833	2.9439
		0.5	1.8034	1.9440	2.0884	2.3876	1.8466	1.9862	2.1296	2.4269
	0.5	0	3.0833	3.3237	3.5706	4.0819	3.1567	3.3953	3.6405	4.1487
		0.1	2.8851	3.1100	3.3411	3.8196	2.9538	3.1771	3.4065	3.8820
		0.3	2.4754	2.6684	2.8666	3.2771	2.5343	2.7259	2.9228	3.3307
		0.5	2.0406	2.1997	2.3632	2.7016	2.0892	2.2472	2.4094	2.7458
	1	0	3.9381	4.2449	4.5600	5.2127	4.0296	4.3342	4.6472	5.2959
		0.1	3.6850	3.9721	4.2670	4.8777	3.7706	4.0556	4.3485	4.9555
		0.3	3.1617	3.4080	3.6610	4.1850	3.2351	3.4797	3.7310	4.2517
		0.5	2.6064	2.8095	3.0180	3.4500	2.6670	2.8686	3.0757	3.5050
Type-X	0	0	2.4057	2.5933	2.7860	3.1851	2.4635	2.6497	2.8410	3.2376
		0.1	2.1954	2.3664	2.5420	2.9057	2.2459	2.4156	2.5901	2.9516
		0.3	1.7741	1.9118	2.0533	2.3463	1.8107	1.9475	2.0881	2.3795
		0.5	1.3514	1.4559	1.5633	1.7856	1.3753	1.4793	1.5860	1.8073
	0.25	0	2.7247	2.9372	3.1555	3.6074	2.7900	3.0009	3.2177	3.6668
		0.1	2.4868	2.6805	2.8794	3.2914	2.5439	2.7362	2.9337	3.3432
		0.3	2.0101	2.1661	2.3264	2.6584	2.0514	2.2065	2.3658	2.6959
		0.5	1.5317	1.6501	1.7718	2.0238	1.5587	1.6765	1.7975	2.0483
	0.5	0	3.0833	3.3237	3.5706	4.0819	3.1567	3.3953	3.6405	4.1487
		0.1	2.8149	3.0341	3.2592	3.7254	2.8791	3.0967	3.3203	3.7838
		0.3	2.2769	2.4536	2.6352	3.0112	2.3235	2.4991	2.6795	3.0535
		0.5	1.7367	1.8709	2.0089	2.2946	1.7672	1.9007	2.0379	2.3223
	1	0	3.9381	4.2449	4.5600	5.2127	4.0296	4.3342	4.6472	5.2959
		0.1	3.5996	3.8797	4.1673	4.7632	3.6798	3.9579	4.2437	4.8360
		0.3	2.9200	3.1465	3.3792	3.8611	2.9786	3.2037	3.4350	3.9143
		0.5	2.2354	2.4082	2.5857	2.9533	2.2742	2.4460	2.6225	2.9884
Type-O	0	0	2.4057	2.5933	2.7860	3.1851	2.4635	2.6497	2.8410	3.2376
		0.1	2.3160	2.4969	2.6827	3.0674	2.3742	2.5537	2.7381	3.1203
		0.3	2.1356	2.3031	2.4751	2.8312	2.1955	2.3615	2.5321	2.8857
		0.5	1.9530	2.1070	2.2652	2.5928	2.0167	2.1692	2.3260	2.6509
	0.25	0	2.7247	2.9372	3.1555	3.6074	2.7900	3.0009	3.2177	3.6668
		0.1	2.6229	2.8278	3.0382	3.4739	2.6886	2.8919	3.1008	3.5336
		0.3	2.4181	2.6076	2.8023	3.2055	2.4857	2.6736	2.8668	3.2671
		0.5	2.2105	2.3848	2.5638	2.9346	2.2823	2.4550	2.6324	3.0001
	0.5	0	3.0833	3.3237	3.5706	4.0819	3.1567	3.3953	3.6405	4.1487
		0.1	2.9672	3.1989	3.4369	3.9297	3.0411	3.2710	3.5072	3.9968
		0.3	2.7335	2.9478	3.1678	3.6235	2.8094	3.0218	3.2401	3.6925
		0.5	2.4962	2.6930	2.8951	3.3136	2.5767	2.7716	2.9719	3.3870
	1	0	3.9381	4.2449	4.5600	5.2127	4.0296	4.3342	4.6472	5.2959
		0.1	3.7858	4.0811	4.3845	5.0127	3.8776	4.1707	4.4719	5.0962
		0.3	3.4778	3.7501	4.0298	4.6089	3.5715	3.8416	4.1191	4.6942
		0.5	3.1631	3.4121	3.6678	4.1974	3.2615	3.5082	3.7617	4.2871

The first three buckling mode shapes for various gradation parameters for simple supported non-porous 2D FG beams were illustrated in Fig. 8. The change in p_z does not cause any significant alteration in buckling mode shapes, whereas an increase in p_x results in a leftward shift of the buckling mode shapes. Fig. 9 illustrates the first three buckling mode shapes for non-porous, Type-H, Type-X, and Type-O. As observed, porosity and its distributions do not significantly affect the buckling mode shapes, with the graphs nearly overlapping at the top of each other. The visibility of changes in the buckling mode shapes becomes more pronounced with an increase in the mode number by both gradation parameters and porosity.

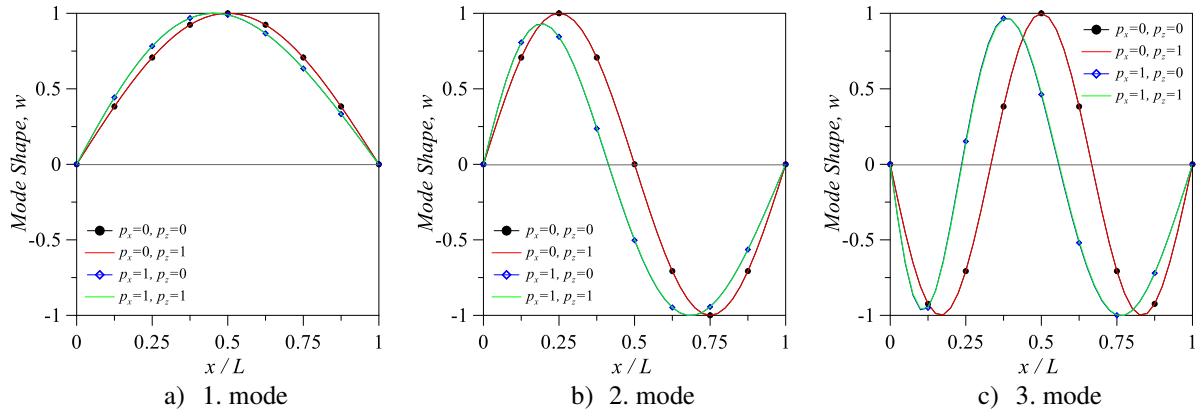


Fig. 8. Buckling mode shapes of the non- porous SS 2D FG beams for various gradation parameters ($L/h=5$)

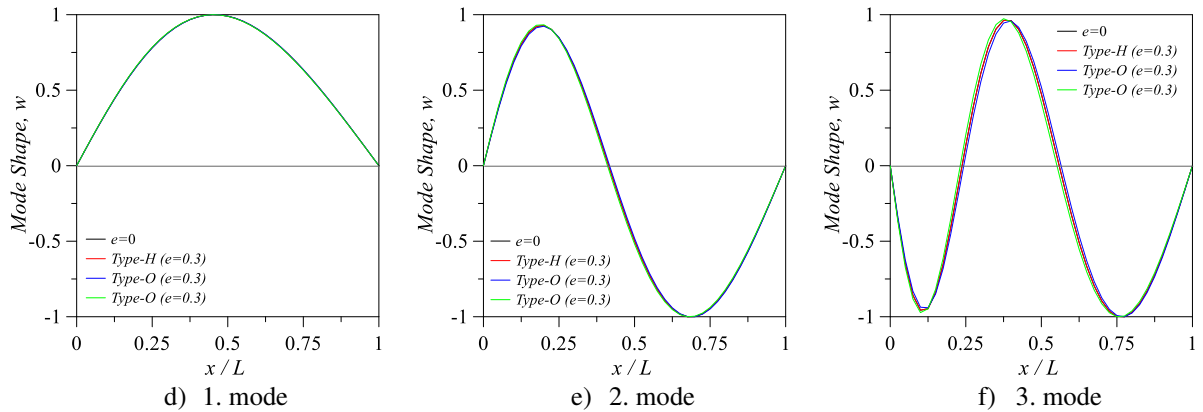


Fig. 9. Buckling mode shapes of the SS 2D FG beams for various porosity distribution ($L/h=5, p_x=p_z=1$)

4. Conclusion

This paper investigates the bending and buckling of porous 2D FG beams. The material properties exhibit exponential variations in both the x and z directions, marking the first instance of such an

application in the static analysis of 2D FG beams. Three different porosity distribution functions throughout the height were considered. The numerical solution was carried out using a finite element comprising 2 nodes and 8 DOFs. After conducting the convergence analysis, it was demonstrated that the results of the current study are consistent with the results reported in the literature. Finally, a comprehensive investigation was conducted on the deflections, stresses, critical buckling loads, and buckling mode shapes of porous 2D FG beams. This analysis considered various factors including porosity, porosity distributions, gradation parameters, slenderness, and boundary conditions, all of which were explored through a meticulous parametric study. Several key conclusions can be drawn from the findings of this study:

- As expected from the higher-order finite element with 2 nodes and 8 DOFs, shear stresses at the top and bottom of the beam were found to be zero, in line with actual results.
- Increasing gradation parameters leads to decreased non-dimensional deflections due to heightened stiffness. Conversely, an increase in porosity has a varied impact on stiffness, resulting in increased deflections.
- Moment of inertia decreases as the porosity shifts away from the neutral axis of the beam. Consequently, the maximum deflections occur for Type-X with porosity concentrated at the top and bottom of the beam, while the minimum deflections are observed for Type-O where porosity is centered around the neutral axis.
- The change in p_z leads to noticeable variations in maximum stresses and stress distributions, while changes in p_x do not cause significant effects.
- Minor differences in stresses are observed between the non-porous beam and the porous beams.
- An increase in gradation parameters leads to higher non-dimensional critical buckling loads, while increasing porosity results in lower critical buckling loads.
- The sequence of critical buckling loads for porosity distributions, from highest to lowest, follows the order of Type-O, Type-H, and Type-X, mirroring their respective moment of inertia.
- The discernibility of alterations in buckling mode shapes becomes more evident with an escalating mode number, influenced by both gradation parameters and porosity.

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