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Mixed series solution for vibration and stability of porous bi-directional functionally graded beams

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Abstract A new analytical solution based on the Ritz method is presented in this paper for analyzing the free vibration and buckling behavior of porous bi-directional functionally graded (2D-FG) beams under various boundary conditions. The solution is based on first-order shear deformation theory (FSDT). The selection of solution functions used in Ritz methods distinguishes the methods from each other and determines the accuracy of the analytical solution. To accurately capture the system's behavior and achieve the desired results, these functions have been carefully selected as a combination of polynomial and trigonometric expressions tailored as mixed series functions for each boundary condition. The study considers three types of porosity, namely PFG-1, PFG-2, and PFG-3. The equations of motion are derived using Lagrange's principle, taking into account the power-law variation of the beam material components throughout the volume. The non-dimensional fundamental frequencies and critical buckling loads are calculated for different boundary conditions, gradation exponents in the x and z directions (p_x , p_z), slenderness (L/h), porosity coefficient (e), and porosity types. Initially, the accuracy of the mixed series functions is investigated for non-porous bi-directional functionally graded beams, and the numerical results are compared with existing literature to validate the proposed solution. Subsequently, the paper focuses on analyzing the influence of porosity on the free vibration and buckling behavior of bi-directional functionally graded beams using the developed solution method.

Keywords Bi-directional functionally graded beam · Porosity · Mixed series functions · Free vibration and buckling · First-order shear deformation theory

1 Introduction

With technological advancements, functionally graded materials (FGMs) have gained significant attention in various engineering applications. FGMs are composed of two or more materials with distinct physical and chemical properties, resulting in a gradual or continuous variation in their structure. These materials are extensively used in various industries, including aerospace, biomedical, military, automotive, power generation, and engineering structures. The continuous variation in properties allows FGMs to efficiently manage thermal stresses, improve thermal insulation, and enhance resistance to mechanical loads and environmental conditions. Compared to homogeneous materials, these materials provide superior strength, stiffness, heat, and corrosion resistance. Bi-directional FGMs, widely researched in recent years, are materials whose composition varies in two directions, providing customizable properties. These properties include thermal management, load-carrying capacity, lightweight design, performance, and flexibility. Bi-directional FGMs offer advantages such as structural integrity, high-temperature performance, resistance to thermal shocks, and optimized structural design. As a result, bi-directional FGMs provide enhanced reliability and extended service life for

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critical engineering structures. Hence, exploring the mechanical behavior of bi-directional functionally graded materials has garnered significant attention from researchers.

The literature focuses on the effect of porosity in functionally graded beams (FGBs). The studies in the literature encompass research investigating this effect, typically carried out using methods such as finite element analysis and analytical solutions. First, the studies investigating the effect of porosity on the mechanical behavior of uni-directional and bi-directional beams using the finite element method have been listed. Numerous studies focus on the effect of porosity in one-directional FGBs with finite element methods. These studies highlight the influence of porous structures on the mechanical behavior and performance of the beams, providing valuable insights into critical parameters such as durability and load-carrying capacity [1–11]. On the other hand, research conducted using finite element methods on the porosity effect of bi-directional functionally graded beams is also noteworthy. Such beams with bi-directional composition properties show to exhibit distinct mechanical behaviors. Chen et al. [12] investigated the linear and nonlinear free vibration analysis of a rotating two-dimensional functionally graded (2D-FG) microbeam with both even and uneven porous distributions. The analysis is based on the Timoshenko beam theory, von Karman geometric nonlinearity assumption, and the modified couple stress theory. The microbeam's material properties vary along the axial and thickness directions. The researchers aim to examine the effects of rotation and porous distributions on the microbeam's vibration behavior, considering the material property variations in different directions. Karamanlı and Aydogdu [13] investigated the free vibration and buckling behaviors of 2D-FG porous microbeams using the modified couple stress theory in their study. They employ a transverse shear normal deformation beam theory and the finite element method for their investigation. The main focus of their research is to examine the effects of various parameters, including the thickness-to-material length scale parameter, micro-porosity volume fraction ratio, boundary condition, aspect ratio, and gradient index, on the dimensionless fundamental frequencies and dimensionless critical buckling loads of the 2D-FG porous microbeams. Karamanlı and Vo [14] presented a finite element model to analyze the structural behaviors of 2D-FG porous microbeams. The model is based on quasi-3D and modified strain gradient theories. The study aims to provide a more comprehensive and precise understanding of the behavior of FG porous microbeams, considering the influence of multiple material length scale parameters to account for size effects. Ramteke and Panda [15] investigated the free vibrational frequencies of a multi-directional FG structure. The novelty of this research lies in considering the influences of variable gradings, such as power-law, sigmoid, and exponential, as well as the effects of porosity distribution (even and uneven type). Turan and Adiyaman [16, 17] presented a new higher-order finite element for the static, free vibration, and buckling analyses of 2D-FG porous beams under various boundary conditions. The method is based on the parabolic shear deformation theory. The main objective of this research is to predict the deflections, stresses, natural frequencies, and critical buckling loads of both porous and non-porous 2D-FG beams using the proposed finite element. Adiyaman [18] conducted a free vibration analysis of a porous 2D-FG beam using a higher-order shear deformation theory. The novel approach employed in this study enables the calculation of natural frequencies even for beams with high porosity parameters. Furthermore, there are various studies in the literature that investigate the mechanical behavior of structures such as one or two-directional porous functionally graded plates and panels using the finite element method. These studies aim to evaluate the effects of porosity on material properties and structural performance from a broader perspective [19–22].

Analytical studies hold great importance in understanding the mechanical behavior of FGBs and ensuring accuracy in the design process. These analytical investigations have captured the interest of researchers as they contribute to revealing the unique properties of FGBs and provide fundamental insights for future engineering applications. Therefore, using analytical methods, numerous studies have emerged that analyze the effect of porosity on one-directional FGBs' static, free vibration, and buckling behaviors [23–33]. As a continuation of these studies, research has been presented that examines the effect of porosity on the mechanical behaviors of bi-directional FGBs using analytical methods. Mirjavadi et al. [34] investigated the thermo-mechanical vibration behavior of a 2D-FG porous nanobeam. The material properties of the nanobeam vary along its thickness and length, following a power law function. The nanobeam is modeled using the Timoshenko beam theory, and the governing equations are derived using Eringen's nonlocal elasticity theory. Shafiei et al. [35] analyzed the vibration behavior of 2D-FG nano and microbeams for the first time, considering that they are made of two different porous materials. The analysis is based on the Timoshenko beam theory. Eringen's nonlocal elasticity theory is utilized for the nano-beams, while the modified couple stress theory is employed for the microbeams. Faroughi et al. [36] investigated the wave propagation of 2D-FG porous rotating nano-beams for the first time. The researchers use the general nonlocal theory and Reddy's beam model to formulate the size-dependent model. Keleshteri and Jelovica [37] investigated the nonlinear vibration behavior of shear deformable bi-directional porous beams with non-uniform porosity distribution. The research

focuses on analyzing the vibration characteristics of these beams and identifying optimal porosity patterns to enhance their natural frequencies. Wang et al. [38], the effects of hygro-thermo-mechanical coupling loadings on the buckling behaviors of a porous 2D-FG Timoshenko nanobeam are investigated through numerical analysis. Bensaid et al. [39] investigated the free vibration and stability response of 2D-FG beams under various boundary conditions. The research aims to provide a comprehensive understanding of the vibrational behavior and stability of 2D-FGBs, considering the combined effects of both normal and shear deformations. Sekkal et al. [40] developed a quasi-3D computational theoretical model to investigate the buckling response of 2D-FG porous beams subjected to variable axial loads. Bridjesh et al. [41] conducted a numerical investigation on the buckling behavior of a porous 2D-FG beam using the higher-order shear deformation theory. The study aimed to gain insights into the buckling characteristics of two-directional porous functionally graded beams and the influence of material variations on their structural stability. Moreover, the Ritz and Navier methods, commonly encountered in the literature, are techniques grounded in analytical principles and widely employed across various engineering and scientific disciplines. These methodologies focus on minimizing the system's energy function to solve the resulting equations. The method used in the present study, being Ritz-based, involves an analytical and mathematical analysis process. Therefore, creating a separate section for studies utilizing the Ritz method and the commonly employed Navier method is deemed necessary. These techniques are considered practical tools in the analytical solution of complex systems and have been applied in numerous fields, including structural analysis, vibration analysis, fluid mechanics, and electromagnetic fields [42–51].

The existing literature indicates a scarcity of studies concerning porous 2D-FG beams, and the reviewed studies primarily focus on nano and microbeams. To the best of the author's knowledge, there has been no investigation into the free vibration and buckling behavior of porous 2D-FG beams, considering different porosity types with the proposed solution. Hence, there is a significant research gap in this specific area that needs to be addressed in the engineering literature. The aim of this paper is to present a new analytical solution for studying the free vibration and buckling behavior of porous 2D-FGBs under different boundary conditions. In pursuit of this objective, new solution functions have been devised utilizing the Ritz method, aiming to yield outcomes in line with existing literature while simultaneously diminishing computational workload and time and simplifying the modeling of intricate problems. Upon reviewing the results, it becomes apparent that our objective has been successfully realized. The solution utilizes a combination of polynomial and trigonometric expressions to create mixed series functions tailored to each boundary condition based on the FSDT. In our calculations, we have opted to use the geometric mid-surface of the 2D-FG beam, a widely adopted approach in the existing literature. The constitutive relations of the beam have been described using Hooke's law. The study considers three types of porosity (PFG-1, PFG-2, and PFG-3) and derives the equations of motion using Lagrange's principle. Non-dimensional fundamental frequencies and critical buckling loads are calculated for various boundary conditions, gradation exponents in the x and z directions (p_x, p_z), slenderness (L/h), porosity coefficient (e), and porosity types.

2 Theoretical formulation for porous 2D-FGBs

2.1 Material properties

The 2D-FG beam shown in Fig. 2 exhibits three different porosity types, and its loading, geometry, and dimensions are provided in Fig. 1. Here, L represents the length of the beam, b represents its width, and h

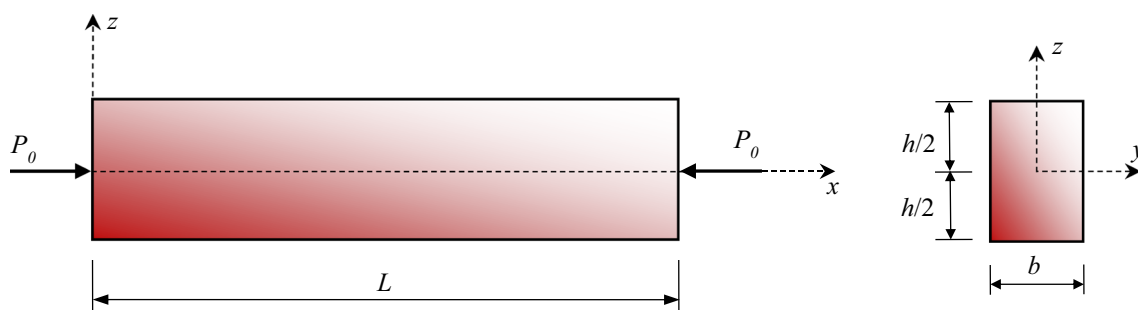


Fig. 1 The porous 2D-FG beam's geometry and applied load

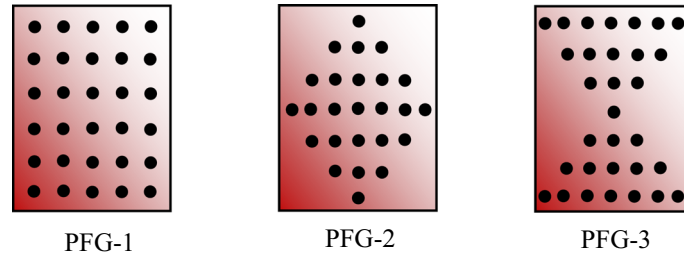


Fig. 2 The beam cross-sections for three types of porosity

represents its height. P_0 denotes the applied compressive force on the beam. The material properties of the beam can be expressed according to the mixture rule, taking into account the porosity as follows.

$$P(x, z) = P_c \left(V_c(x, z) - \frac{e(z)}{2} \right) + P_m \left(V_m(x, z) - \frac{e(z)}{2} \right) \quad (1)$$

Here, $P(x, z)$ represents Young's modulus $E(x, z)$, shear modulus $G(x, z)$, and mass density $\rho(x, z)$ of the beam. Furthermore, P_c , P_m , and $e(x, z)$ denote the material properties of ceramic and metal and porosity functions, respectively. The relationship between the volume fractions of metal and ceramic is defined in Eq. (2), while Eq. (3) illustrates the volume fraction of ceramic.

$$V_c(x, z) + V_m(x, z) = 1 \quad (2)$$

$$V_c(x, z) = (x/L)^{p_x} (0.5 + z/h)^{p_z} \quad (3)$$

The gradation exponents of material properties are represented by p_x and p_z . The effective material properties can be determined by using Eqs. (1)–(3) in the following manner.

$$P(x, z) = P_m + (P_c - P_m)(x/L)^{p_x} (0.5 + z/h)^{p_z} - e(z) \frac{(P_c + P_m)}{2} \quad (4)$$

The function $e(z)$ represents the distribution of porosity along the thickness, and in this study, three types of porosity distribution functions, namely PFG-1, PFG-2, and PFG-3, are defined as follows.

$$e_1(z) = e, \quad e_2(z) = e \left(1 - \frac{2|z|}{h} \right), \quad e_3(z) = e \left(\frac{2|z|}{h} \right) \quad (5)$$

where e signifies the porosity coefficient, defined as the ratio between the volume of voids and the total volume.

For PFG-1, porosities were evenly distributed across the cross-section of the beam. In contrast, for PFG-2 and PFG-3, porosities were predominantly concentrated in the center and corners of the section, respectively. Figure 3 illustrates the variation in Young's modulus $E(x, z)$ and mass density $\rho(x, z)$ for these three porosity types, considering different gradation exponents and porosity coefficients. Comparing the E and ρ of the porous 2D-FG beam with the non-porous beam, it is evident that the material properties deteriorate as porosity increases.

2.2 Mathematical formulation

By the first-order shear deformation beam theory (FSDT), the displacements at any point along the beam, excluding those on the neutral axis, can be expressed as follows:

$$U(x, z, t) = u(x, t) + z \phi(x, t), \quad W(x, z, t) = w(x, t) \quad (6)$$

where U and W represent the displacements along the x (horizontal) and z (transverse) axes, respectively. Under the assumptions of geometric and physical linearity, the strain–displacement relations and stresses of the beam can be expressed as follows:

$$\varepsilon_{xx} = \frac{\partial U}{\partial x} = u_{,x} + z \phi_{,x}, \quad \gamma_{xz} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} = w_{,x} + \phi \quad (7)$$

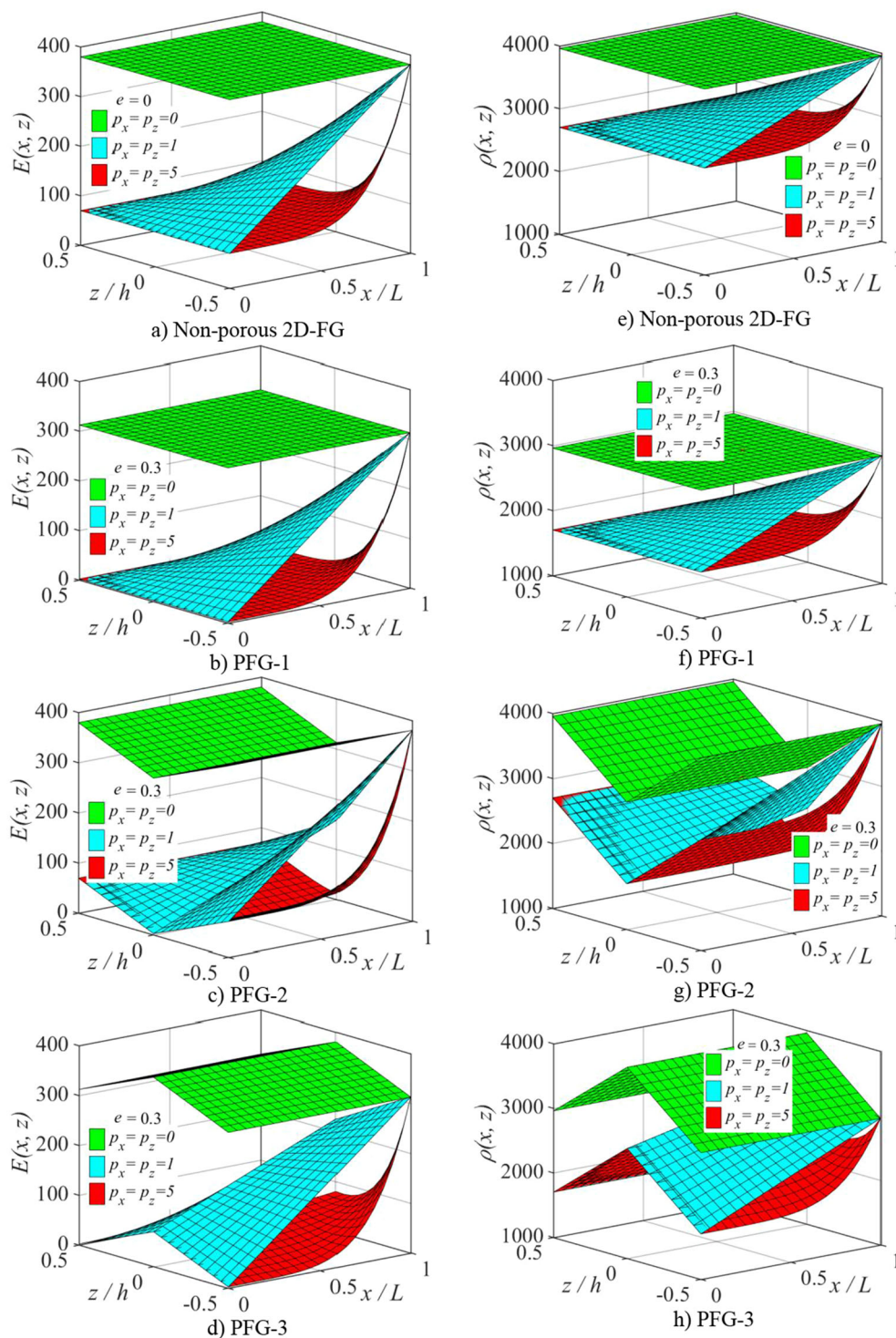


Fig. 3 The change in E and ρ with respect to p_x, p_z , and e

$$\sigma_{xx} = E(x, z)\varepsilon_{xx}, \quad \tau_{xz} = K G(x, z)\gamma_{xz} \quad (8)$$

In this context, ε_{xx} , γ_{xz} , σ_{xx} , τ_{xz} , and K signify the normal strain, shear strains, normal stress, shear stress, and the shear correction factor, respectively. The symbol $(\cdot)_{,x}$ signifies the derivative with respect to x . For rectangular cross-sections, the shear correction coefficient K is set to 5/6. The equations of motion can be derived using Lagrange's equations as follows:

$$\frac{d}{dt} \left(\frac{\partial \Pi}{\partial \dot{q}_i} \right) - \frac{\partial \Pi}{\partial q_i} = 0 \quad (9)$$

where the symbol Π denotes the Lagrangian functional, wherein q_i represents the variables of u_i , w_i , and ϕ_i . The Lagrangian functional itself is defined as a sum of three components, namely, $\Pi = T - (U^* + V)$, where T represents the kinetic energy, U^* signifies the strain energy, and V accounts for the work done by external loads.

The strain energy of the beam resulting from deformation can be expressed as follows, where A represents the beam's cross-sectional area.

$$\begin{aligned} U^* &= \frac{1}{2} \int_0^L \int_A (\sigma_{xx}\varepsilon_{xx} + \tau_{xz}\gamma_{xz}) dA dx \\ &= \frac{1}{2} \int_0^L \int_A \left(\left((E_c - E_m)(x/L)^{p_x} (0.5 + z/h)^{p_z} + E_m - e(z) \frac{(E_c + E_m)}{2} \right) ((u_{,x})^2 + 2zu_{,x}\phi_{,x} + z^2(\phi_{,x})^2) \right. \\ &\quad \left. + K \left((G_c - G_m)(x/L)^{p_x} (0.5 + z/h)^{p_z} + G_m - e(z) \frac{(G_c + G_m)}{2} \right) ((w_{,x})^2 + 2w_{,x}\phi + \phi^2) \right) dA dx \quad (10) \\ &= \frac{1}{2} \int_0^L \left((u_{,x})^2 \{A_0(x/L)^{p_x} + A_1\} + 2u_{,x}\phi_{,x} \{B_0(x/L)^{p_x} + B_1\} + (\phi_{,x})^2 \{C_0(x/L)^{p_x} + C_1\} \right. \\ &\quad \left. + (w_{,x})^2 \{D_0(x/L)^{p_x} + D_1\} + 2w_{,x}\phi \{D_0(x/L)^{p_x} + D_1\} + \phi^2 \{D_0(x/L)^{p_x} + D_1\} \right) dx \end{aligned}$$

The stiffness coefficients here are defined as follows.

$$\begin{aligned} [A_0, B_0, C_0] &= b \int_{-h/2}^{h/2} (E_c - E_m)(0.5 + z/h)^{p_z} [1, z, z^2] dz, \\ [A_1, B_1, C_1] &= b \int_{-h/2}^{h/2} \left(E_m - e(z) \frac{(E_c + E_m)}{2} \right) [1, z, z^2] dz \\ D_0 &= b \int_{-h/2}^{h/2} K (G_c - G_m)(0.5 + z/h)^{p_z} dz, \\ D_1 &= b \int_{-h/2}^{h/2} K \left(G_m - e(z) \frac{(E_c + E_m)}{2} \right) dz \end{aligned} \quad (11)$$

The beam's kinetic energy can be expressed as follows, with the dot indicating the derivative with respect to time.

$$\begin{aligned} T &= \frac{1}{2} \int_0^L \int_A \rho(x, z) (\dot{U}^2 + \dot{W}^2) dA dx \\ &= \frac{1}{2} \int_0^L \int_A \left((\rho_c - \rho_m)(x/L)^{p_x} (0.5 + z/h)^{p_z} + \rho_m - e(z) \frac{(\rho_c + \rho_m)}{2} \right) (\dot{u}^2 + 2z\dot{u}\dot{\phi} + z^2\dot{\phi}^2 + \dot{w}^2) dA dx \\ &= \frac{1}{2} \int_0^L \left((\dot{u})^2 \{I_0(x/L)^{p_x} + I_{00}\} + 2\dot{u}\dot{\phi} \{I_1(x/L)^{p_x} + I_{11}\} + (\dot{\phi})^2 \{I_2(x/L)^{p_x} + I_{22}\} \right. \\ &\quad \left. + (\dot{w})^2 \{I_0(x/L)^{p_x} + I_{00}\} \right) dx \end{aligned} \quad (12)$$

Table 1 Kinematic boundary conditions and mixed functions employed in the analytical solution

BCs	ψ_i	$x = 0$	$x = L$
C–C	$((L-x)\sin(\frac{x}{L}i))^2$	$u = 0, w = 0, \phi = 0, w' = 0$	$u = 0, w = 0, \phi = 0, w' = 0$
S–S	$x(L-x)(\sin(\frac{x}{L}i) + \cos(\frac{x}{L}i))$	$u = 0, w = 0$	$w = 0$
C–F	$x \sin(\frac{x}{L}i)$	$u = 0, w = 0, \phi = 0, w' = 0$	

The inertia coefficients here are defined as follows.

$$[I_0, I_1, I_2] = b \int_{-h/2}^{h/2} (\rho_c - \rho_m)(0.5 + z/h)^{P_z} [1, z, z^2] dz, \quad (13)$$

$$[I_{00}, I_{11}, I_{22}] = b \int_{-h/2}^{h/2} \left(\rho_m - e(z) \frac{(\rho_c + \rho_m)}{2} \right) [1, z, z^2] dz$$

The expression for the work done by the pressure force, P_0 , acting along the axis of the beam at its ends, is as follows:

$$V = \frac{1}{2} \int_0^L P_0(w_{,x})^2 dx \quad (14)$$

The displacements to be utilized in the above equations for the numerical solution can be considered as follows:

$$u(x, t) = \sum_{i=1}^m \psi_{i,x}(x) u_i(t), w(x, t) = \sum_{i=1}^m \psi_i(x) w_i(t), \phi(x, t) = \sum_{i=1}^m \psi_{i,x}(x) \phi_i(t) \quad (15)$$

Here, $\psi_i(x)$ and $\psi_{i,x}(x)$ denote mixed series functions, which vary depending on the beam's boundary conditions. The generalized nodal displacements are represented by $u_i(t)$, $w_i(t)$, and $\phi_i(t)$, and 'm' corresponds to the number of mixed series functions. The specific mixed series functions chosen to meet the boundary conditions for the beams analyzed in this study are detailed in Table 1. In this context, C–C designates the clamped–clamped beam, S–S stands for the simply supported beam, and C–F indicates the clamped-free beam.

The work and energy expressions are incorporated into Lagrange's equations, taking into account the solution functions outlined in Table 1. Consequently, the equation of motion for a beam with a length of L can be derived as follows:

$$\mathbf{M}\ddot{\mathbf{X}} + (\mathbf{K}_e - P_0\mathbf{K}_g)\mathbf{X} = \mathbf{F} \quad (16)$$

In this context, $\mathbf{M}$, $\mathbf{K}_e$, $\mathbf{K}_g$, and $\mathbf{F}$ represent the system's mass, stiffness, geometric stiffness matrices, and load vector. The matrices here are explicitly expressed as follows for the porous C–F 2D-FG beam.

$$\mathbf{M} = \begin{bmatrix} \mathbf{m}^{11} & \mathbf{0} & \mathbf{m}^{13} \\ & \mathbf{m}^{22} & \mathbf{0} \\ sym. & & \mathbf{m}^{33} \end{bmatrix}_{3m \times 3m}, \mathbf{K}_e = \begin{bmatrix} \mathbf{k}^{11} & \mathbf{0} & \mathbf{k}^{13} \\ & \mathbf{k}^{22} & \mathbf{k}^{23} \\ sym. & & \mathbf{k}^{33} \end{bmatrix}_{3m \times 3m}, \mathbf{K}_g = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & g^{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}_{3m \times 3m},$$

$$X_1 = \left\{ \sin\left(\frac{x}{L}i\right) \sin\left(\frac{x}{L}j\right) + i \frac{x}{L} \cos\left(\frac{x}{L}i\right) \sin\left(\frac{x}{L}j\right) + j \frac{x}{L} \sin\left(\frac{x}{L}i\right) \cos\left(\frac{x}{L}j\right) + ij \frac{x^2}{L^2} \cos\left(\frac{x}{L}i\right) \cos\left(\frac{x}{L}j\right) \right\},$$

$$X_2 = \left\{ 4 \cos\left(\frac{x}{L}i\right) \cos\left(\frac{x}{L}j\right) - 2j \frac{x}{L} \cos\left(\frac{x}{L}i\right) \sin\left(\frac{x}{L}j\right) - 2i \frac{x}{L} \sin\left(\frac{x}{L}i\right) \cos\left(\frac{x}{L}j\right) + ij \frac{x^2}{L^2} \sin\left(\frac{x}{L}i\right) \sin\left(\frac{x}{L}j\right) \right\},$$

$$\mathbf{m}_{ij}^{11} = \int_0^L \left\{ I_0 \left(\frac{x}{L} \right)^{P_x} + I_{00} \right\} X_1 dx, \quad \mathbf{m}_{ij}^{13} = \int_0^L \left\{ I_1 \left(\frac{x}{L} \right)^{P_x} + I_{11} \right\} X_1 dx,$$

$$\mathbf{m}_{ij}^{22} = \int_0^L \left\{ I_0 \left(\frac{x}{L} \right)^{P_x} + I_{00} \right\} \left\{ x^2 \sin\left(\frac{x}{L}i\right) \sin\left(\frac{x}{L}j\right) \right\} dx,$$

$$\begin{aligned}
\mathbf{m}_{ij}^{33} &= \int_0^L \left\{ I_2 \left(\frac{x}{L} \right)^{p_x} + I_{22} \right\} X_1 dx, \\
\mathbf{k}_{ij}^{11} &= \int_0^L \frac{ij}{L^2} \left\{ A_0 \left(\frac{x}{L} \right)^{p_x} + A_1 \right\} X_2 dx, \\
\mathbf{k}_{ij}^{13} &= \int_0^L \frac{ij}{L^2} \left\{ B_0 \left(\frac{x}{L} \right)^{p_x} + B_1 \right\} X_2 dx, \\
\mathbf{k}_{ij}^{22} &= \mathbf{k}_{ij}^{23} = \int_0^L \left\{ D_0 \left(\frac{x}{L} \right)^{p_x} + D_1 \right\} X_1 dx, \\
\mathbf{k}_{ij}^{33} &= \int_0^L \frac{ij}{L^2} \left\{ C_0 \left(\frac{x}{L} \right)^{p_x} + C_1 \right\} X_2 dx + \int_0^L \left\{ D_0 \left(\frac{x}{L} \right)^{p_x} + D_1 \right\} X_1 dx, \\
\mathbf{g}_{ij}^{22} &= \int_0^L X_1 dx, \quad \text{with } i = j = 1, 2, \dots, m
\end{aligned} \tag{17}$$

The generalized displacement vector $\mathbf{u}$ can be defined as follows:

$$\mathbf{u} = \{u_1 \quad \dots \quad u_m \quad w_1 \quad \dots \quad w_m \quad \phi_1 \quad \dots \quad \phi_m\}^T \tag{18}$$

For the free vibration analysis of the beam, when we set $\mathbf{F} = \mathbf{0}$ and $P_0 = 0$ in Eq. (16) and consider $\mathbf{X} = \mathbf{u}e^{i\omega t}$ as the solution, the equation takes the following form:

$$(\mathbf{K}_e - \omega^2 \mathbf{M})\mathbf{u} = \mathbf{0} \tag{19}$$

Here, ω represents the natural frequency of the beam.

For the buckling analysis of the beam, when we set $\mathbf{M} = \mathbf{0}$ and $\mathbf{F} = \mathbf{0}$ in Eq. (16) and consider $\mathbf{X} = \mathbf{u}$ as the solution, the equation can be expressed as follows:

$$(\mathbf{K}_e - P_0 \mathbf{K}_g)\mathbf{u} = \mathbf{0} \tag{20}$$

The Eqs. (19) and (20) represent standard eigenvalue problems. In these equations, the parameters ω and $P_0 = (P_0)_{cr}$ correspond to the natural frequencies and critical buckling loads of the beam, respectively. The solution to these eigenvalue problems involves finding the values of ω and P_0 that nullify the coefficient matrices, thus yielding the eigenvalues (natural frequencies and buckling loads) associated with the system's vibrational and buckling characteristics.

3 Results and discussion

This section provides the numerical results obtained through the utilization of MATLAB [54] for the free vibration and buckling analysis of porous 2D-FGBs. The analyses encompass three distinct boundary conditions: simply supported (S–S), clamped–clamped (C–C), and clamped–free (C–F). Within these investigations, the fundamental frequencies and critical buckling loads are determined across a spectrum of parameters, encompassing gradation exponents, porosity coefficients, porosity distributions, and slenderness ratios. The shear correction factor, represented as K , is established at $5/6$ for rectangular cross-sections. Regarding the material properties of the metal (Al) and ceramic (Al_2O_3) employed in 2D-FGBs, they are defined as $E_m = 70$ GPa, $\rho_m = 2702$ kg/m³, $\nu = 0.3$ for the metal, and $E_c = 380$ GPa, $\rho_c = 3960$ kg/m³, $\nu = 0.3$ for the ceramic. It's important to note that the fundamental frequencies and critical buckling loads presented in the results are non-dimensional and are expressed as follows:

$$\bar{\omega} = \omega \left(\frac{L^2}{h} \sqrt{\frac{\rho_m}{E_m}} \right), \quad \bar{\lambda} = (P_0)_{cr} \left(\frac{12L^2}{E_m b h^3} \right) \tag{21}$$

Table 2 Variation of non-dimensional fundamental frequencies and critical buckling loads with the number of series ($L/h = 5$, $p_x = p_z = 1$, $e = 0.1$)

Number of series (m)	PFG-1			PFG-2			PFG-3		
	S-S	C-C	C-F	S-S	C-C	C-F	S-S	C-C	C-F
<i>Dimensionless fundamental frequency</i>									
1	3.7188	6.6786	1.1598	3.8623	6.9628	1.2337	3.7202	6.7169	1.1685
2	3.3800	6.6268	1.0555	3.5332	6.9280	1.1304	3.3886	6.6707	1.0659
3	3.3279	6.5215	0.9901	3.4889	6.8214	1.0696	3.3399	6.5722	1.0026
4	3.3227	6.5165	0.9727	3.4833	6.8185	1.0545	3.3349	6.5683	0.9862
5	3.2956	6.4744	0.9642	3.4612	6.7751	1.0473	3.3098	6.5288	0.9783
6	3.2956	6.4721	0.9596	3.4612	6.7736	1.0436	3.3097	6.5270	0.9742
8	3.2951	6.4483	0.9553	3.4609	6.7494	1.0401	3.3093	6.5049	0.9703
10	3.2949	6.4336	0.9550	3.4607	6.7344	1.0396	3.3091	6.4912	0.9700
<i>Dimensionless critical buckling load</i>									
1	17.0716	44.2766	3.1183	19.5532	51.0926	3.7328	20.2933	47.5445	3.3459
2	14.1810	41.2923	2.9015	16.4632	48.3681	3.5147	15.0133	44.6461	3.1248
3	13.5867	40.8818	2.7799	15.8998	47.9917	3.4046	14.9255	44.2992	3.0042
4	13.5436	40.8269	2.7452	15.8480	47.9325	3.3758	14.6529	44.2493	2.9709
5	13.2004	40.7803	2.7290	15.5425	47.8810	3.3631	14.5949	44.2072	2.9557
6	13.1999	40.7650	2.7207	15.5421	47.8547	3.3568	14.5096	44.1874	2.9481
8	13.1922	40.7419	2.7131	15.5362	47.8207	3.3513	14.4312	44.1610	2.9412
10	13.1889	40.7261	2.7106	15.5339	47.7992	3.3499	14.3816	44.1442	2.9398

where the subscript " m " indicates properties associated with the metal component. As deduced from Eq. (1) (when $e = 0$), it's noteworthy that opting for gradation exponents $p_x = p_z = 0$ imparts ceramic characteristics to the beam's material. Conversely, as the gradation exponents increase, the material progressively takes on metal-like properties.

3.1 Convergence evaluation

Conducting a convergence study becomes essential to ascertain the appropriate number of series utilized within the solution. Table 2 exhibits the fluctuations in non-dimensional fundamental frequencies and critical buckling loads in correlation with the number of series (m) for different boundary conditions and porosity types. It's evident from the table that convergence is achieved when m is set to 6. Subsequently, the analysis is carried out with this selected number of series in the ensuing results.

3.2 Validation assessment

The validation assessment for the new proposed mixed series solution has been conducted concerning non-porous 2D-FGBs. Table 3 compares non-dimensional fundamental frequencies between the current study and a previous investigation [52], encompassing various gradation exponents, slenderness ratios, and boundary conditions. In their study, Karamanli [52] employed the Ritz method based on a four-unknown shear and normal deformable beam theory. Furthermore, to compare the outcomes of the present study, non-dimensional critical buckling loads have been evaluated based on another former investigation [53] conducted by the same author. The results of this comparison, involving various gradation exponents, slenderness ratios, and boundary conditions, are outlined in Table 4. In their research, Karamanli [53] utilized an analytical solution grounded on a third-order shear deformable beam theory. An observation of Tables 3 and 4 demonstrates a high degree of compatibility between the results obtained. When the gradation exponents, p_x and p_z , are heightened, the stiffness of the beam diminishes. Consequently, both the fundamental frequencies and buckling loads decrease as a result. Conversely, an increase in slenderness leads to an upsurge in fundamental frequencies and buckling loads. However, it's worth noting that while the actual values decrease, the dimensionless results increase due to the non-dimensionalization process. As expected, the highest fundamental frequencies and buckling loads are attained under the clamped-clamped (C-C) boundary condition. At the same time, the lowest values are observed for the clamped-free (C-F) boundary condition.

Table 3 Comparison of the non-dimensional fundamental frequencies of the non-porous bi-directional functionally graded beams

BCs	Theory	p_z	$L/h = 5$				$L/h = 20$			
			p_x				p_x			
			0	1	2	5	0	1	2	5
S-S	Present	0	5.1525	4.1734	3.5713	2.9589	5.4603	4.4324	3.7660	3.1026
	Ref. [52]		5.1616	4.1791	3.5772	2.9643	5.4610	4.4310	3.7660	3.1030
	Present	1	3.9902	3.4626	3.1632	2.8408	4.2050	3.6628	3.3350	2.9867
	Ref. [52]		4.0073	3.4744	3.1727	2.8468	4.2345	3.6762	3.3414	2.9881
	Present	2	3.6344	3.2806	3.0623	2.8099	3.8368	3.4762	3.2356	2.9605
	Ref. [52]		3.6434	3.2888	3.0685	2.8132	3.8763	3.4936	3.2435	2.9620
C-C	Present	5	3.4312	3.1546	2.9783	2.7773	3.6509	3.3582	3.1591	2.9353
	Ref. [52]		3.4126	3.1504	2.9758	2.7758	3.6822	3.3703	3.1642	2.9360
	Present	0	10.1201	8.0136	7.4138	6.6974	12.2346	9.6340	8.9086	8.1031
	Ref. [52]		10.1759	8.0146	7.4211	6.7044	12.2721	9.6508	8.9312	8.1208
	Present	1	8.0226	6.8461	6.4978	6.1064	9.4439	8.1215	7.7567	7.3441
	Ref. [52]		8.0672	6.8189	6.4827	6.0929	9.5300	8.1285	7.7478	7.3283
C-F	Present	2	7.3004	6.4579	6.2196	5.9231	8.6155	7.7330	7.4615	7.1415
	Ref. [52]		7.2947	6.4068	6.1621	5.8831	8.7146	7.7202	7.4274	7.1056
	Present	5	6.7414	6.1571	5.9622	5.7028	8.1825	7.4749	7.2384	6.9679
	Ref. [52]		6.5887	6.0187	5.8589	5.6831	8.2446	7.4470	7.1994	6.9363
	Present	0	1.8986	1.1015	0.9492	0.9102	1.9506	1.1236	0.9694	0.9322
	Ref. [52]		1.9053	1.0953	0.9526	0.9134	1.9531	1.1137	0.9703	0.9333
C-F	Present	1	1.4656	1.0516	0.9686	0.9460	1.5018	1.0747	0.9912	0.9701
	Ref. [52]		1.4803	1.0534	0.9724	0.9493	1.5141	1.0736	0.9927	0.9713
	Present	2	1.3361	1.0472	0.9802	0.9595	1.3704	1.0720	1.0043	0.9846
	Ref. [52]		1.3523	1.0491	0.9837	0.9628	1.3863	1.0722	1.0059	0.9858
	Present	5	1.2670	1.0465	0.9928	0.9737	1.3044	1.0740	1.0188	0.9998
	Ref. [52]		1.2762	1.0474	0.9956	0.9769	1.3176	1.0746	1.0203	1.0010

Table 4 Comparison of the non-dimensional critical buckling loads of the non-porous bi-directional functionally graded beams

BCs	Theory	p_z	$L/h = 5$				$L/h = 20$			
			p_x				p_x			
			0	1	2	5	0	1	2	5
S-S	Present	0	48.5904	25.6458	17.0382	11.0890	53.2364	28.6091	18.6885	11.9380
	Ref. [53]		48.5959	24.9833	16.6297	10.9883	53.2364	28.5105	18.6436	11.9306
	Present	1	24.5814	16.4833	13.0745	10.1873	26.5619	18.0541	14.2278	10.9988
	Ref. [53]		24.5838	16.3126	12.9394	10.1423	26.5619	18.0342	14.2161	10.9954
	Present	2	19.1617	14.3984	12.1037	9.9375	20.7253	15.7780	13.2007	10.7669
	Ref. [53]		19.0709	14.2731	12.0082	9.8965	20.7185	15.7645	13.1927	10.7638
C-C	Present	5	15.9417	12.8794	11.2625	9.6717	17.5073	14.2298	12.3651	10.5382
	Ref. [53]		15.6435	12.7414	11.1712	9.6200	17.4842	14.2169	12.3578	10.5342
	Present	0	152.1223	74.7420	54.3659	39.5159	209.3098	105.2145	78.1972	57.2851
	Ref. [53]		152.1470	72.0959	52.0853	38.8730	208.9510	104.9660	77.3658	56.6534
	Present	1	79.6582	51.1869	42.5883	34.8341	104.8285	69.5248	59.0023	49.0981
	Ref. [53]		79.4835	50.0389	41.2629	34.3748	104.5630	68.7433	57.8242	48.3525
C-F	Present	2	61.8285	44.6681	38.9789	33.3455	81.6952	61.3745	54.0921	47.0010
	Ref. [53]		60.8783	43.4884	37.7746	32.8623	81.4649	60.2068	52.8695	46.0630
	Present	5	49.6407	39.4170	35.4800	31.9395	68.7914	55.1831	50.2148	44.9794
	Ref. [53]		46.8870	37.7663	34.5080	31.4285	68.3268	54.3477	49.2052	44.2740
	Present	0	13.0593	4.7949	3.3264	2.6053	13.3730	4.8587	3.3685	2.6445
	Ref. [53]		13.0595	4.7498	3.3263	2.6053	13.3730	4.8128	3.3684	2.6445
C-F	Present	1	6.5352	3.5779	2.9107	2.5223	6.6673	3.6312	2.9542	2.5659
	Ref. [53]		6.5352	3.5691	2.9107	2.5223	6.6673	3.6222	2.9542	2.5659
	Present	2	5.0981	3.2971	2.8059	2.4998	5.2025	3.3525	2.8530	2.5464
	Ref. [53]		5.0916	3.2877	2.8032	2.4983	5.2021	3.3464	2.8529	2.5464
	Present	5	4.2925	3.0695	2.7068	2.4753	4.3984	3.1325	2.7608	2.5266
	Ref. [53]		4.2704	3.0557	2.6993	2.4711	4.3970	3.1291	2.7603	2.5264

3.3 Free vibration of porous 2D-FGBs

Both tables and figures present non-dimensional fundamental frequencies for porous 2D-FGBs according to various parameters. Additionally, vibration mode shapes are included in the analysis. Tables 5, 6 and 7, categorized by boundary conditions (S–S, C–C, and C–F), offer insights into non-dimensional fundamental frequencies concerning various gradation exponents, slenderness ratios, porosity coefficients, and porosity types. As observed in these tables, an increase in p_x and p_z leads to decreased frequencies similar to non-porous beams. A direct proportionality exists between slenderness and fundamental frequencies, meaning they either increase or fall together. However, the impact of the porosity coefficient is contingent on gradation exponents and the porosity type. The impact of the porosity coefficient on the alteration of fundamental frequencies becomes more pronounced as gradation exponents increase due to the resulting decrease in rigidity. For smaller gradation exponent values, the type of porosity has a relatively minimal effect on the change in fundamental frequencies. Moreover, if the material is transitioning toward a more metallic state, the most influential porosity type in terms of fundamental frequency alteration is PFG-1. To illustrate this effect, consider the case of C–C boundary conditions with $p_x = p_z = 5$ and $L/h = 5$. As the porosity coefficient escalates from 0.1 to 0.3, the dimensionless fundamental frequency substantially decreases from 5.1463 to 1.9587. This trend is particularly prominent in PFG-1 compared to PFG-2 and PFG-3, where the total void volume within the beam is relatively higher. The variation in fundamental frequencies is more pronounced in PFG-3 when contrasted with PFG-2. Even though the total void volumes in PFG-2 and PFG-3 are relatively similar, their distribution differs significantly. In PFG-2, the voids are primarily concentrated towards the center of the cross-section, whereas in PFG-3, the voids become more pronounced as they approach the top and bottom surfaces. This observation underscores the significance of both the total void volume and the spatial distribution of voids as critical factors influencing the fundamental frequencies.

Figure 4 illustrates the variation in non-dimensional fundamental frequencies of S–S, C–C, and C–F 2D-FGBs with three distinct porosity distributions according to the porosity coefficient and different gradation exponents. Moreover, the non-dimensional fundamental frequencies of porous S–S supported 2D-FGBs have been represented in 3D diagrams in Fig. 5, showcasing the variation based on the gradation exponent, porous distribution, and porosity coefficient to enhance visual clarity. Due to the similarity in graph variations across other boundary conditions, only results for a S–S supported beam are presented. Observing the figures, it's evident that as the porosity coefficient increases, the natural frequencies consistently decrease across all three porosity scenarios. Nevertheless, for PFG-2, this trend doesn't hold for values of p_x and p_z less than 1. The impact of porosity becomes more pronounced with higher values of p_x and p_z . Figure 6 compares non-dimensional natural frequencies among four types of beams: non-porous, PFG-1, PFG-2, and PFG-3, based on L/h values for $p_x = p_z = 1$ and $e = 0.3$. The non-porous beam and PFG-2 results closely align, and they exhibit the highest natural frequency. In contrast, the lowest natural frequency is observed in PFG-1. Furthermore, as slenderness increases, the non-dimensional natural frequency rises until it reaches a specific value, which remains constant. When the cross-section of a beam decreases, or in other words, when its slenderness increases, its natural frequency decreases due to a stability issue. As the cross-section narrows, its resistance to bending decreases, making the beam more prone to bending. This results in the beam becoming more flexible and slender, leading to a reduction in its natural frequency. Therefore, the reduction in the beam's cross-section leads to a decrease in its natural frequency. However, our values increase due to the non-dimensionalization process, as natural frequency values are multiplied by L^2 in the non-dimensionalization process.

Figure 7 shows the first three vibration mode shapes of porous 2D-FGBs for different boundary conditions, porosity coefficients, and PFG-1. Upon careful examination of the figure, it is apparent that the vibration mode shapes share comparable distributions, albeit with slight discrepancies in the locations of maximum displacements. Despite S–S and C–C displaying symmetrical geometries, their mode shapes do not exhibit symmetrical behavior, primarily due to the asymmetrical material properties. Additionally, these mode shapes offer valuable insights into the structural response of the beams under different conditions, shedding light on the influence of porosity and boundary constraints on their vibrational characteristics.

3.4 Buckling of porous 2D-FGBs

Critical buckling loads and buckling mode shapes for porous 2D-FGBs are presented using tables and figures. Tables 8, 9 and 10 present non-dimensional critical buckling loads for various parameters, including L/h , p_x , p_z , e , and porosity types under the scenarios of S–S, C–C, and C–F boundary conditions. An examination of these tables reveals that an increase in gradation exponents and porosity coefficient corresponds to a decrease

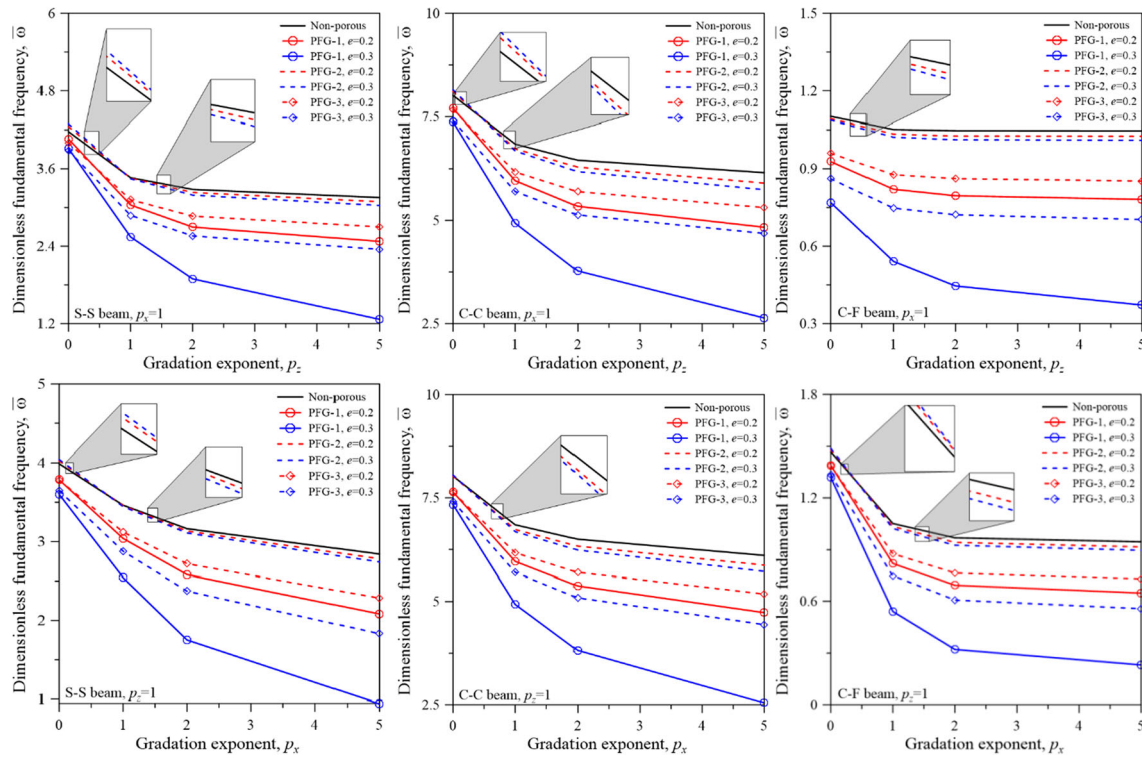


Fig. 4 Variation of non-dimensional fundamental frequencies of porous 2D-FGBs with various boundary conditions according to the gradation exponent and porosity coefficient ($L/h = 5$)

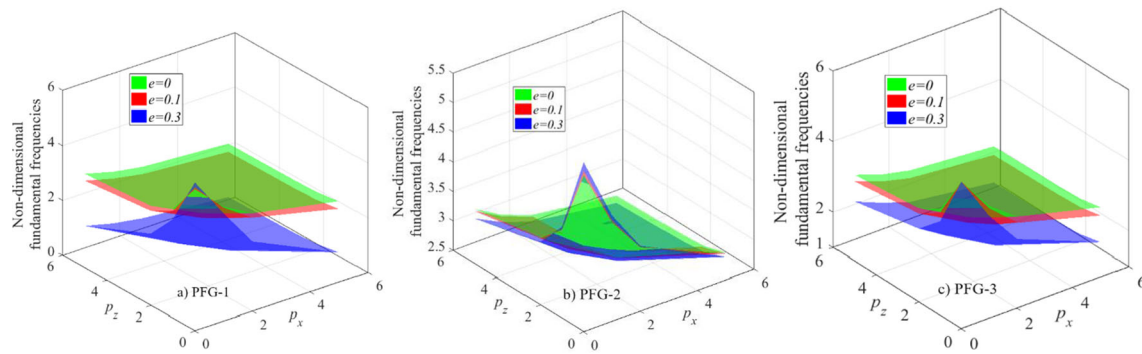


Fig. 5 Non-dimensional fundamental frequencies of porous S-S 2D-FGBs for **a** PFG-1, **b** PFG-2, and **c** PFG-3 ($L/h = 5$)

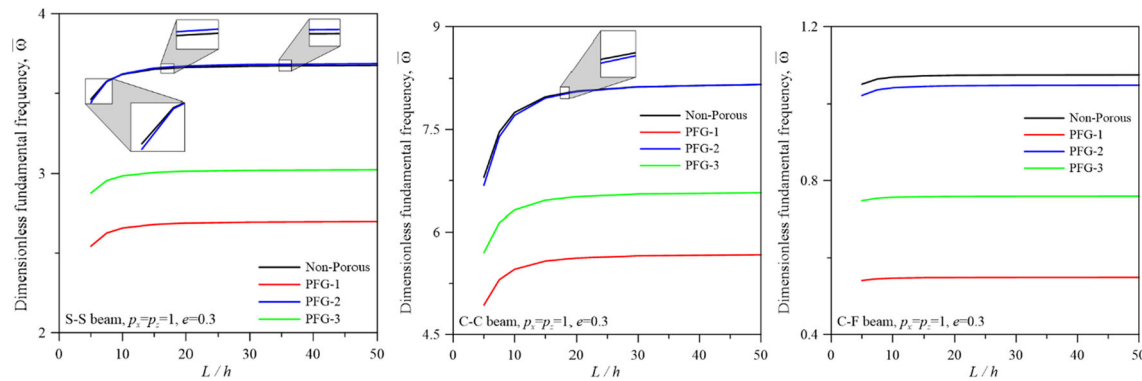


Fig. 6 Variation of non-dimensional fundamental frequencies of porous 2D-FGBs with various boundary conditions according to the L/h

Table 5 The non-dimensional fundamental frequencies of the S-S porous bi-directional functionally graded beams

Type	e	p_z	$L/h = 5$				$L/h = 20$			
			p_x				p_x			
			0	1	2	5	0	1	2	5
PFG-1	0.1	0	5.2221	4.1304	3.4154	2.6834	5.5341	4.3906	3.5997	2.8087
	0.2		5.3045	4.0609	3.1648	2.2366	5.6215	4.3243	3.3340	2.3344
	0.3		5.4038	3.9162	2.5456	1.0933	5.7267	4.1919	2.6870	1.1334
	0.1	1	3.9069	3.2956	2.9376	2.5499	4.1117	3.4843	3.0935	2.6759
	0.2		3.7864	3.0391	2.5801	2.0843	3.7864	3.2110	2.7120	2.1803
	0.3		3.6015	2.5437	1.7521	0.9427	3.7731	2.6890	1.8373	0.9773
	0.1	2	3.4496	3.0585	2.8096	2.5131	3.6340	3.2383	2.9652	2.6435
	0.2		3.1554	2.6965	2.4011	2.0383	3.3128	2.8499	2.5285	2.1373
	0.3		2.6174	1.8935	1.4099	0.8819	2.7313	1.9923	1.4765	0.9153
PFG-2	0.1	5	3.1816	2.9041	2.7099	2.4757	3.3803	3.0923	2.8737	2.6137
	0.2		2.7295	2.4753	2.2707	1.9941	2.8882	2.6337	2.4053	2.1004
	0.3		1.5271	1.2656	1.0980	0.8219	1.5957	1.3325	1.1535	0.8564
	0.1	0	5.2202	4.2109	3.5815	2.9383	5.5370	4.4812	3.7838	3.0869
	0.2		5.2931	4.2507	3.5909	2.9131	5.6196	4.5344	3.8027	3.0682
	0.3		5.3715	4.2927	3.5989	2.8814	5.7087	4.5927	3.8225	3.0457
	0.1	1	4.0085	3.4612	3.1504	2.8137	4.2277	3.6686	3.3285	2.9650
	0.2		4.0264	3.4561	3.1329	2.7807	4.2503	3.6721	3.3189	2.9394
	0.3		4.0435	3.4454	3.1084	2.7394	4.2722	3.6717	3.3045	2.9085
PFG-3	0.1	2	3.6187	3.2615	3.0403	2.7801	3.8235	3.4637	3.2202	2.9368
	0.2		3.5949	3.2337	3.0110	2.7437	3.8016	3.4435	3.1991	2.9088
	0.3		3.5596	3.1928	2.9704	2.6976	3.7675	3.4115	3.1691	2.8745
	0.1	5	3.3966	3.1278	2.9515	2.7451	3.6208	3.3407	3.1412	2.9107
	0.2		3.3456	3.0894	2.9160	2.7055	3.5735	3.3132	3.1168	2.8815
	0.3		3.2694	3.0319	2.8663	2.6548	3.4994	3.2690	3.0820	2.8459
	0.1	0	5.1511	4.0958	3.4170	2.7257	5.4540	4.3444	3.5946	2.8479
	0.2		5.1495	4.0020	3.2187	2.4191	5.4470	4.2387	3.3757	2.5157
	0.3		5.1477	3.8835	2.9421	1.9732	5.4394	4.1061	3.0730	2.0384
	0.1	1	3.8945	3.3097	2.9679	2.5996	4.0958	3.4923	3.1191	2.7221
	0.2		3.7819	3.1210	2.7183	2.2845	3.9682	3.2832	2.8450	2.3795
	0.3		3.6473	2.8774	2.3739	1.8321	3.8168	3.0151	2.4706	1.8936
	0.1	2	3.4825	3.0975	2.8524	2.5660	3.6665	3.2725	3.0033	2.6923
	0.2		3.2953	2.8666	2.5830	2.2477	3.4581	3.0172	2.7073	2.3451
	0.3		3.0573	2.5602	2.2108	1.7920	3.1958	2.6810	2.3022	1.8544
	0.1	5	3.2446	2.9545	2.7589	2.5312	3.4420	3.1357	2.9160	2.6639
	0.2		3.0045	2.6989	2.4766	2.2104	3.1749	2.8526	2.6048	2.3126
	0.3		2.6786	2.3514	2.0842	1.7518	2.8157	2.4705	2.1761	1.8164

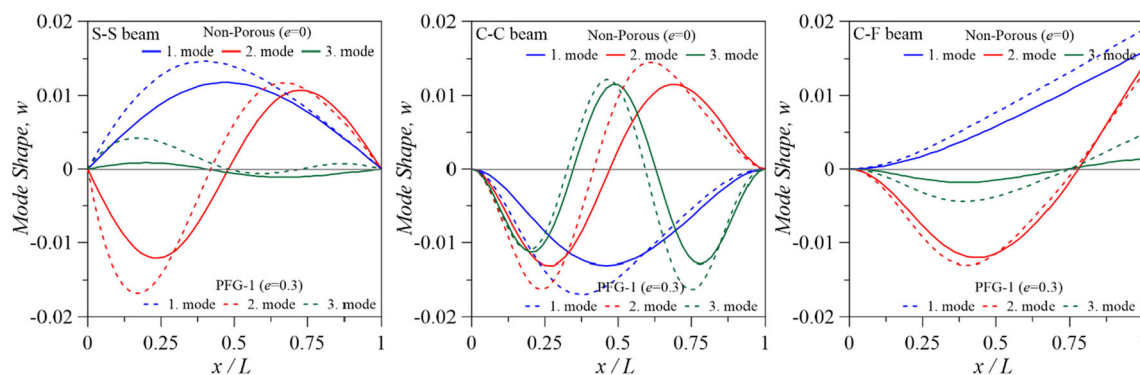
**Fig. 7** Vibration mode shapes of the 2D-FGBs for various boundary conditions ($L/h = 5$, $p_x = p_z = 2$)

Table 6 The non-dimensional fundamental frequencies of the C–C porous bi-directional functionally graded beams

Type	e	p_z	$L/h = 5$				$L/h = 20$			
			p_x				p_x			
			0	1	2	5	0	1	2	5
PFG-1	0.1	0	10.2397	7.8985	7.0874	6.2000	12.3205	9.4159	8.4915	7.5135
	0.2		10.4014	7.7039	6.6122	5.4222	12.5151	9.2170	7.8933	6.5972
	0.3		10.5961	7.3775	5.6111	3.1909	12.7494	8.7093	6.6115	3.9511
	0.1	1	7.8341	6.4721	6.0407	5.5606	9.1734	7.6369	7.1869	6.6837
	0.2		7.6397	5.9621	5.3775	4.7339	8.8790	6.9696	6.3547	5.6868
	0.3		7.3341	4.9322	3.8184	2.5472	8.42962	5.6226	4.3852	3.0797
	0.1	2	6.9359	6.0148	5.7059	5.3502	8.1091	7.1371	6.8205	6.4450
	0.2		6.4301	5.3413	4.9439	4.4851	7.4011	6.2556	5.8588	5.3909
	0.3		5.4676	3.7750	3.0846	2.2709	6.1148	4.2308	3.5076	2.7139
	0.1	5	6.2762	5.6318	5.4050	5.1463	7.5287	6.8208	6.5580	6.2487
	0.2		5.4925	4.8348	4.5635	4.2453	6.4445	5.7962	5.5091	5.1532
	0.3		3.2728	2.6345	2.3637	1.9587	3.5796	2.9357	2.6906	2.3317
	0.1	0	10.2200	8.0651	7.3666	6.5899	12.3251	9.6733	8.8946	8.0529
	0.2		10.3454	8.1058	7.3528	6.5047	12.5067	9.7661	8.9448	8.0550
	0.3		10.4804	8.1471	7.3255	6.3791	12.7026	9.8671	8.9964	8.0497
PFG-2	0.1	1	7.9953	6.7736	6.3997	5.9806	9.4274	8.0688	7.6922	7.2645
	0.2		8.0235	6.7331	6.3303	5.8715	9.4766	8.0670	7.6745	7.2268
	0.3		8.0503	6.6811	6.2357	5.7207	9.5245	8.0554	7.6451	7.1746
	0.1	2	7.2153	6.3708	6.0992	5.7852	8.5251	7.6462	7.3735	7.0481
	0.2		7.1674	6.2904	5.9993	5.6599	8.4761	7.5971	7.3230	6.9912
	0.3		7.1000	6.1827	5.8649	5.4890	8.4001	7.5225	7.2497	6.9139
	0.1	5	6.6326	6.0237	5.8219	5.5932	8.0562	7.3769	7.1418	6.8681
	0.2		6.5217	5.9052	5.6932	5.4508	7.9493	7.3089	7.0776	6.8017
	0.3		6.3676	5.7442	5.5182	5.2565	7.7831	7.2041	6.9834	6.7109
	0.1	0	10.1166	7.8667	7.1156	6.2997	12.1444	9.3419	8.4733	7.5537
	0.2		10.1307	7.6687	6.7758	5.8274	12.1312	9.0403	7.9841	6.8955
	0.3		10.1455	7.4125	6.2846	5.1116	12.1166	8.6572	7.2888	5.9118
	0.1	1	7.8143	6.5270	6.1250	5.6834	9.1385	7.6663	7.2391	6.7622
	0.2		7.6323	6.1722	5.6987	5.1819	8.8585	7.1692	6.6442	6.0701
	0.3		7.4107	5.6999	5.0903	4.4453	8.5260	6.5144	5.8143	5.0758
PFG-3	0.1	2	6.9986	6.1106	5.8204	5.4885	8.1813	7.2177	6.9068	6.5423
	0.2		6.6837	5.6976	5.3581	4.9727	7.7229	6.6356	6.2625	5.8297
	0.3		6.2736	5.1316	4.7028	4.2238	7.1446	5.8583	5.3709	4.8185
	0.1	5	6.4043	5.7630	5.5453	5.2993	7.6669	6.9233	6.6595	6.3558
	0.2		6.0013	5.3112	5.0590	4.7727	7.0800	6.2855	5.9798	5.6260
	0.3		5.4425	4.6849	4.3712	4.0132	6.2888	5.4179	5.0361	4.5949

in critical buckling loads. Conversely, higher slenderness ratios lead to a rise in critical buckling loads. Notably, the impact of the porosity coefficient on critical buckling loads becomes more pronounced with increasing values of p_x and p_z . The influence of porosity within the beam is most pronounced in FGP-1 and FGP-3, with FGP-1 showing a slightly more pronounced effect due to a higher presence of voids within the cross-section. A common feature of the void distribution in both FGP-1 and FGP-3 is the concentration of voids near the upper and lower regions of the cross-section. Conversely, in FGP-2, compared to other porosity types, porosity has a relatively lesser impact on buckling loads. This is attributed to the voids primarily located in the center of the cross-section rather than the upper and lower regions. The variation in critical buckling loads behaves similarly across different porosity coefficients for all boundary conditions.

The graphs in Fig. 8 illustrate the variation in non-dimensional critical buckling loads of porous 2D-FGBs according to the porosity coefficient and types. These graphs are presented for different porosity distributions, boundary conditions, and gradient exponents. Furthermore, the variation of non-dimensional critical buckling loads of porous simply supported 2D-FGBs according to the gradation exponent and porosity coefficient (for PFG-1, PFG-2, and PFG-3) has been visualized using three-dimensional diagrams in Fig. 9 for better visual observation. Since the variations of the graphs are similar according to other boundary conditions, results are only provided for a simply supported beam. An increase in the porosity coefficient leads to a reduction in critical buckling loads across all values of p_x and p_z . This decrease in critical buckling loads is attributed to the loss in stiffness caused by porosity.

Table 7 The non-dimensional fundamental frequencies of the C–F porous bi-directional functionally graded beams

Type	e	p_z	$L/h = 5$				$L/h = 20$			
			p_x				p_x			
			0	1	2	5	0	1	2	5
PFG-1	0.1	0	1.9242	1.0292	0.8463	0.7950	1.9769	1.0487	0.8633	0.8136
	0.2		1.9546	0.9278	0.6913	0.6189	2.0081	0.9438	0.7036	0.6327
	0.3		1.9912	0.7695	0.3764	0.2244	2.0457	0.7806	0.3808	0.2284
	0.1	1	1.4338	0.9596	0.8597	0.8291	1.4684	0.9795	0.8788	0.8497
	0.2		1.3881	0.8208	0.6921	0.6473	1.4203	0.8361	0.7061	0.6629
	0.3		1.3183	0.5412	0.3209	0.2320	1.3472	0.5486	0.3251	0.2367
	0.1	2	1.2666	0.9487	0.8694	0.8422	1.2978	0.9702	0.8900	0.8639
	0.2		1.1563	0.7956	0.6968	0.6585	1.1829	0.8119	0.7121	0.6749
	0.3		0.9557	0.4458	0.3051	0.2350	0.9750	0.4523	0.3095	0.2400
	0.1	5	1.1738	0.9440	0.8804	0.8560	1.2076	0.9682	0.9030	0.8787
	0.2		1.0045	0.7817	0.7035	0.6703	1.0317	0.8007	0.7208	0.6878
	0.3		0.5578	0.3720	0.2917	0.2382	0.5696	0.3789	0.2971	0.2437
PFG-2	0.1	0	1.9245	1.0973	0.9370	0.8953	1.9781	1.1198	0.9576	0.9182
	0.2		1.9523	1.0925	0.9230	0.8781	2.0076	1.1154	0.9444	0.9024
	0.3		1.9824	1.0870	0.9069	0.8578	2.0396	1.1104	0.9292	0.8844
	0.1	1	1.4729	1.0436	0.9565	0.9323	1.5099	1.0672	0.9799	0.9576
	0.2		1.4802	1.0340	0.9424	0.9163	1.5180	1.0583	0.9669	0.9434
	0.3		1.4872	1.0223	0.9257	0.8970	1.5259	1.0474	0.9518	0.9270
	0.1	2	1.3310	1.0380	0.9683	0.9464	1.3657	1.0636	0.9934	0.9728
	0.2		1.3228	1.0266	0.9543	0.9309	1.3579	1.0532	0.9808	0.9593
	0.3		1.3104	1.0122	0.9373	0.9121	1.3458	1.0400	0.9659	0.9436
	0.1	5	1.2555	1.0372	0.9814	0.9611	1.2938	1.0659	1.0087	0.9887
	0.2		1.2380	1.0257	0.9677	0.9462	1.2770	1.0560	0.9970	0.9760
	0.3		1.2111	1.0105	0.9509	0.9280	1.2506	1.0431	0.9830	0.9612
PFG-3	0.1	0	1.8971	1.0376	0.8656	0.8180	1.9482	1.0569	0.8823	0.8360
	0.2		1.8955	0.9600	0.7594	0.7000	1.9457	0.9761	0.7722	0.7134
	0.3		1.8938	0.8615	0.6127	0.5348	1.9429	0.8741	0.6208	0.5426
	0.1	1	1.4288	0.9742	0.8797	0.8513	1.4626	0.9938	0.8983	0.8712
	0.2		1.3856	0.8771	0.7658	0.7293	1.4169	0.8928	0.7800	0.7441
	0.3		1.3343	0.7475	0.6068	0.5570	1.3627	0.7586	0.6156	0.5657
	0.1	2	1.2783	0.9654	0.8895	0.8641	1.3094	0.9864	0.9094	0.8848
	0.2		1.2073	0.8619	0.7729	0.7407	1.2348	0.8785	0.7880	0.7562
	0.3		1.1176	0.7218	0.6094	0.5659	1.1410	0.7332	0.6187	0.5750
	0.1	5	1.1961	0.9613	0.9004	0.8775	1.2296	0.9846	0.9220	0.8991
	0.2		1.1051	0.8526	0.7810	0.7527	1.1340	0.8710	0.7974	0.7689
	0.3		0.9822	0.7033	0.6127	0.5752	1.0055	0.7158	0.6229	0.5848

Figure 10 presents the variation of non-dimensional critical buckling loads for porous 2D-FGBs under different boundary conditions based on L/h . Notably, the non-porous beam exhibits the highest critical buckling load, while the lowest is observed in PFG-1. Furthermore, as the slenderness increases, the non-dimensional critical buckling load rises to a specific threshold and stabilizes. But it should be that when the cross-section of a beam narrows, i.e., when its slenderness increases, the critical buckling load decreases because the beam becomes more flexible and consequently has a lower load-bearing capacity. As the beam's cross-section narrows, its bending resistance decreases, reducing the critical buckling load. A thinner cross-section allows the beam to buckle more quickly, decreasing the critical buckling load. Therefore, as the cross-section of a beam narrows, its slenderness increases and the critical buckling load decreases. However, due to the non-dimensionalization process, our values increase because the critical buckling load is multiplied by L^2 in the non-dimensionalization process.

Figure 11 illustrates the first three buckling mode shapes for porous 2D-FGBs (PFG-1) under various boundary conditions and porosity coefficients. Observing the figure, we can draw similar conclusions to those made for vibration mode shapes. There are differences in the magnitude and location of maximum values in the distribution of buckling mode shapes. These mode shapes lack symmetry due to their non-symmetrical material properties.

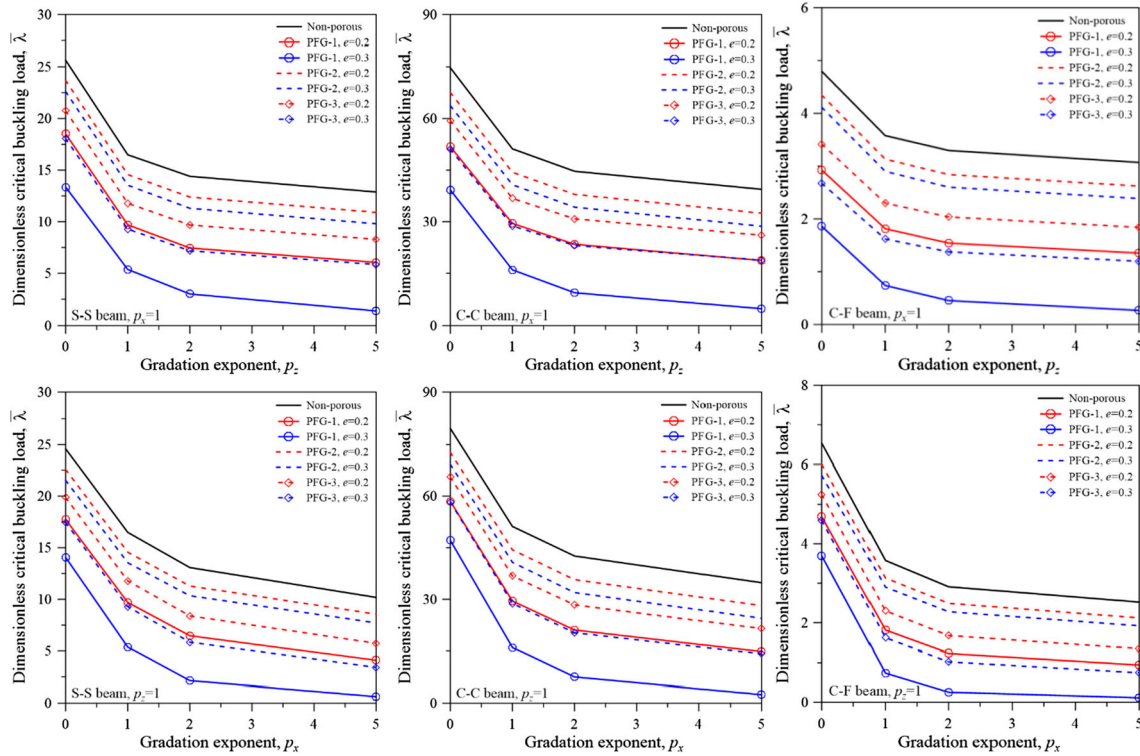


Fig. 8 Variation of non-dimensional critical buckling loads of porous 2D-FGBs with various boundary conditions according to the gradation exponent and porosity coefficient ($L/h = 5$)

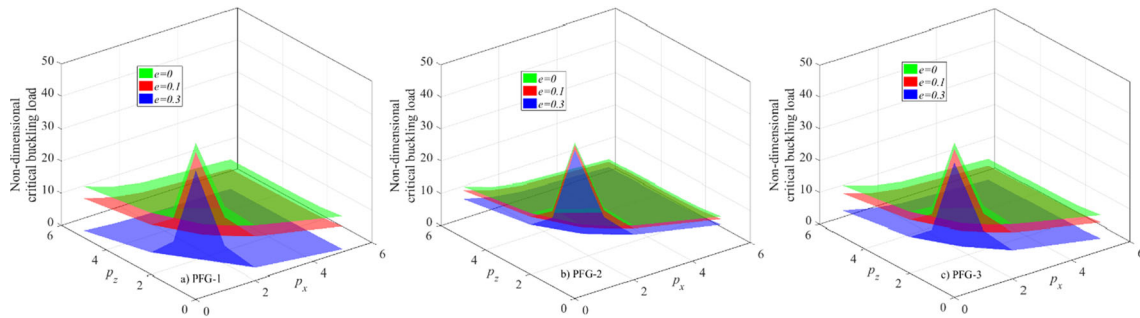


Fig. 9 Non-dimensional critical buckling loads of porous S-S 2D-FGBs for **a**) PFG-1, **b**) PFG-2, and **c**) PFG-3 ($L/h = 5$)

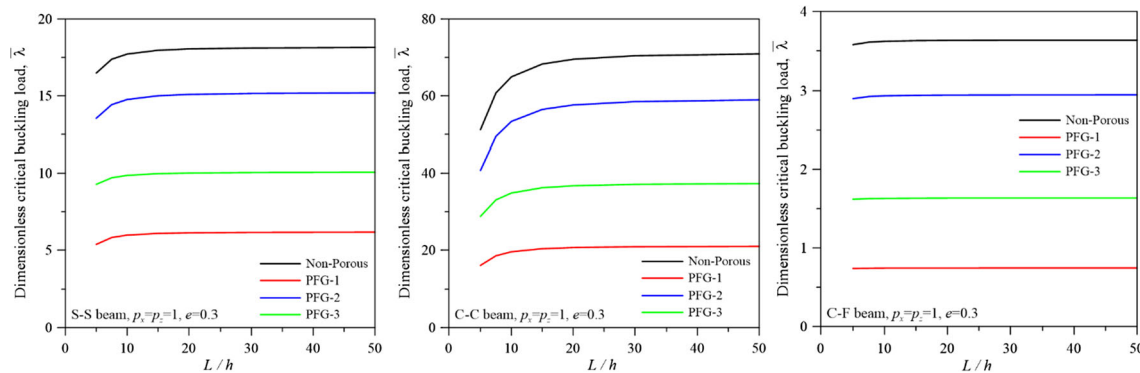


Fig. 10 Variation of non-dimensional critical buckling loads of porous 2D-FGBs with various boundary conditions according to the L/h

Table 8 The non-dimensional critical buckling loads of the S-S porous bi-directional functionally graded beams

Type	e	p_z	$L/h = 5$				$L/h = 20$			
			p_x				p_x			
			0	1	2	5	0	1	2	5
PFG-1	0.1	0	45.7134	22.2500	13.5607	7.9482	50.0843	25.0236	14.9361	8.5435
	0.2		42.8363	18.5143	9.7315	4.6580	46.9321	21.2202	10.8399	5.0006
	0.3		39.9592	13.3511	3.8671	0.8080	43.7799	16.6912	4.9750	0.8738
	0.1	1	21.2225	13.1999	9.8775	7.1805	22.8575	14.4677	10.7450	7.7323
	0.2		17.7356	9.6938	6.4761	4.0880	19.0142	10.6569	7.0523	4.3854
	0.3		14.0603	5.3739	2.1309	0.6354	14.9725	6.1261	2.4362	0.6801
	0.1	2	15.4497	11.0605	8.9417	6.9604	16.6109	12.1013	9.7388	7.5219
	0.2		11.4189	7.4530	5.5948	3.9137	12.1571	8.1265	6.0788	4.2110
	0.3		6.8301	2.9981	1.4863	0.5739	7.1448	3.2686	1.6272	0.6121
	0.1	5	12.1800	9.6195	8.1785	6.7315	13.2945	10.6252	8.9770	7.3220
	0.2		7.8641	6.0581	4.9314	3.7397	8.4561	6.6638	5.4031	4.0537
	0.3		2.1416	1.3573	0.9644	0.5121	2.2010	1.4328	1.0301	0.5475
PFG-2	0.1	0	47.8033	24.6881	16.1004	10.2572	52.4433	27.7169	17.7643	11.0940
	0.2		47.0142	23.6881	15.1300	9.4094	51.6500	26.8150	16.8268	10.2442
	0.3		46.2229	22.6215	14.1093	8.5356	50.8565	25.9012	15.8731	9.3873
	0.1	1	23.5877	15.5421	12.1955	9.3810	25.5089	17.0973	13.3348	10.1776
	0.2		22.5688	14.5663	11.2908	8.5608	24.4268	16.1127	12.4246	9.3513
	0.3		21.5203	13.5412	10.3475	7.7173	23.3105	15.0912	11.4904	8.5177
	0.1	2	18.0087	13.4185	11.2194	9.1326	19.4863	14.7655	12.2972	9.9476
	0.2		16.7986	12.3919	10.3049	8.3127	18.1805	13.7062	11.3691	9.1223
	0.3		15.5179	11.2992	9.3451	7.4678	16.7919	12.5813	10.4050	8.2879
	0.1	5	14.7567	11.9189	10.3938	8.8697	16.2378	13.2473	11.4853	9.7261
	0.2		13.4814	10.9039	9.4930	8.0516	14.8602	12.2106	10.5809	8.9087
	0.3		12.0825	9.8072	8.5413	7.2060	13.3331	11.0903	9.6370	8.0822
PFG-3	0.1	0	46.4987	23.2467	14.5364	8.7908	50.8772	25.9430	15.9024	9.4048
	0.2		44.4049	20.7476	11.8905	6.4161	48.5179	23.1812	12.9632	6.8055
	0.3		42.3090	18.0609	8.9680	3.8920	46.1584	20.2393	9.7273	4.0736
	0.1	1	22.2395	14.5096	10.7758	7.9887	23.9394	15.4581	11.6645	8.5625
	0.2		19.8620	11.7855	8.3953	5.7414	21.2869	12.7972	9.0247	6.0926
	0.3		17.4416	9.2843	5.8535	3.3973	18.5993	10.0214	6.2295	3.5511
	0.1	2	16.6599	12.0764	9.8439	7.7649	17.9167	13.1612	10.6694	8.3487
	0.2		14.0816	9.6856	7.5182	5.5511	15.0398	10.4803	8.0805	5.9052
	0.3		11.4046	7.1828	5.0676	3.2553	12.0780	7.6964	5.3802	3.4074
	0.1	5	13.4519	10.6162	9.0601	7.5299	14.6717	11.6600	9.8813	8.1396
	0.2		10.8507	8.2886	6.8048	5.3539	11.7264	9.0278	7.3524	5.7225
	0.3		8.0868	5.8535	4.4480	3.1089	8.6285	6.2957	4.7369	3.2647

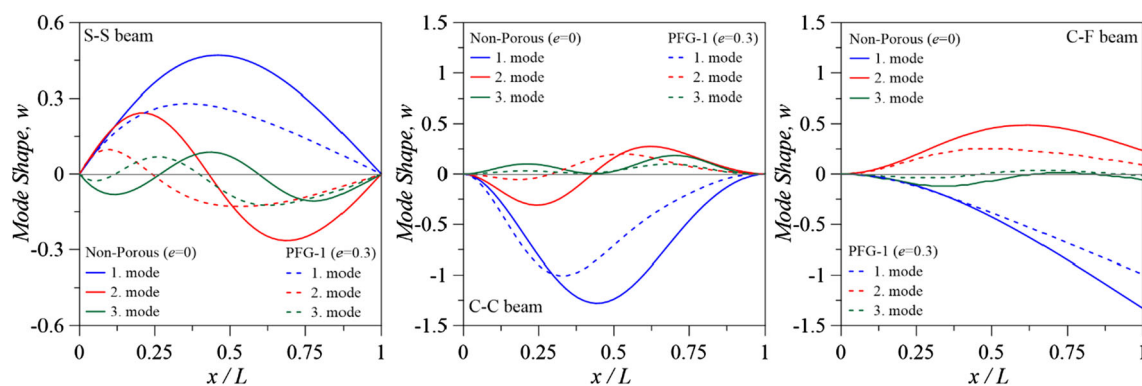
**Fig. 11** Buckling mode shapes of the 2D-FGBs for various boundary conditions ($L/h = 5$, $p_x = p_z = 2$)

Table 9 The non-dimensional critical buckling loads of the C–C porous bi-directional functionally graded beams

Type	e	p_z	$L/h = 5$				$L/h = 20$			
			p_x				p_x			
			0	1	2	5	0	1	2	5
PFG-1	0.1	0	143.1196	63.6192	42.6055	28.9360	197.0736	89.6856	62.2063	42.7047
	0.2		134.1121	51.8244	29.8543	17.3338	184.6704	73.3460	44.3645	26.8033
	0.3		125.1046	39.2437	15.8187	3.3746	172.2671	55.4872	23.3104	5.8745
	0.1	1	69.1811	40.7650	32.2731	25.0630	90.2715	55.0956	44.8603	35.6075
	0.2		58.4694	29.5094	21.0835	14.8140	75.1671	39.5565	29.5798	21.4482
	0.3		47.1414	16.0095	7.4408	2.4735	59.2741	20.6598	10.1727	4.1151
	0.1	2	50.5774	34.5583	29.0468	23.8163	65.6265	46.8335	40.3349	33.6993
	0.2		38.3059	23.4430	18.3413	13.8575	48.1306	31.1314	25.4174	19.8598
	0.3		23.9903	9.3555	5.1717	2.2136	28.3915	11.4570	6.8341	3.5644
	0.1	5	38.4662	29.4376	26.0085	22.5316	52.3541	41.3038	36.8392	31.9712
	0.2		25.7378	18.7338	15.7921	12.8535	33.4076	25.8288	22.3820	18.4682
	0.3		7.9025	4.7151	3.4308	1.9412	8.7783	5.5276	4.3840	2.9991
PFG-2	0.1	0	149.2106	71.2298	50.5059	35.9568	206.2970	101.1650	74.2127	53.5896
	0.2		146.2791	67.5375	46.4295	32.1680	203.1146	97.2614	70.1350	49.8238
	0.3		143.3315	63.7197	42.0616	27.9857	199.9295	93.2954	65.9433	45.9580
	0.1	1	76.1365	47.8547	39.2338	31.5781	100.6551	65.6773	55.3135	45.6143
	0.2		72.7063	44.3700	35.7194	28.1592	96.3685	61.7204	51.5352	42.0703
	0.3		69.1975	40.7464	31.9572	24.4781	91.9489	57.6176	47.6344	38.4357
	0.1	2	58.0508	41.4820	35.7554	30.2015	76.8814	57.3450	50.5588	43.5279
	0.2		54.1250	37.9516	32.2910	26.8352	71.7266	53.1664	46.7133	39.9901
	0.3		50.0159	34.2377	28.6013	23.2214	66.2503	48.7469	42.6920	36.3453
	0.1	5	45.7471	36.0296	32.4307	28.7751	63.8334	51.5715	46.7923	41.6224
	0.2		41.6308	32.4733	28.9937	25.4566	58.3958	47.4687	43.0424	38.1466
	0.3		37.2175	28.6855	25.3170	21.8920	52.3821	43.0551	39.0892	34.5575
PFG-3	0.1	0	146.0220	67.2684	46.5746	32.5454	200.2512	93.6830	66.4397	46.5365
	0.2		139.9015	59.3855	38.2614	25.1703	191.0231	81.8366	53.7902	35.2294
	0.3		133.7640	51.0501	29.1366	16.8808	181.7922	69.2890	39.7542	22.6602
	0.1	1	72.5944	44.1874	35.6840	28.3028	94.5557	59.0911	48.6819	39.1595
	0.2		65.5324	36.7799	28.3850	21.5114	84.1574	48.2695	37.9113	28.9382
	0.3		58.2722	28.7881	20.3032	14.1430	73.6103	36.7633	26.2075	18.1174
	0.1	2	54.5041	38.0168	32.3898	26.9977	70.7814	51.0128	44.1823	37.2241
	0.2		46.8610	30.8389	25.3985	20.3602	59.5033	40.3619	33.7635	27.2386
	0.3		38.7763	23.1052	17.7762	13.2332	47.8707	29.1811	22.6996	16.8075
	0.1	5	42.5562	32.8721	29.2869	25.6530	57.7873	45.3849	40.5688	35.4219
	0.2		35.0818	26.0682	22.6323	19.1776	46.2778	35.0238	30.4838	25.6519
	0.3		26.9792	18.7975	15.4842	12.2833	34.1448	24.2068	19.9170	15.5444

4 Conclusion

The free vibration and buckling behavior analysis in both non-porous and porous 2D-FGBs is conducted utilizing newly developed mixed series functions tailored to specific boundary conditions. The selection of solution functions used in Ritz methods distinguishes the methods from each other and determines the accuracy of the analytical solution. To accurately capture the system's behavior and achieve the desired results, these functions have been carefully selected as a combination of polynomial and trigonometric expressions tailored as mixed series functions for each boundary condition. This key aspect sets this study apart from others. The selection of functions in the Ritz method has significantly contributed to the accuracy of the results and facilitated the computational process. The analytical solution yields non-dimensional fundamental frequencies, vibration mode shapes, non-dimensional critical buckling loads, and buckling mode shapes. The study incorporates three distinct porosity distribution types. Following a comprehensive parametric investigation involving diverse boundary conditions, gradation exponents, porosity coefficients, porosity types, and slenderness ratios, the following conclusions emerge:

- The aim was to design new solution functions based on the Ritz method to obtain results consistent with the existing literature while also reducing computational workload and time and simplifying the modeling of complex problems. Looking at the results, it can be seen that our goal has been successfully achieved.

Table 10 The non-dimensional critical buckling loads of the C–F porous bi-directional functionally graded beams

Type	e	p_z	$L/h = 5$				$L/h = 20$			
			p_x				p_x			
			0	1	2	5	0	1	2	5
PFG-1	0.1	0	12.2861	3.8872	2.4533	1.8065	12.5812	3.9345	2.4812	1.8314
	0.2		11.5128	2.9246	1.5228	0.9931	11.7894	2.9557	1.5372	1.0049
	0.3		10.7396	1.8599	0.4286	0.1258	10.9976	1.8754	0.4307	0.1267
	0.1	1	5.6276	2.7207	2.0885	1.7366	5.7365	2.7570	2.1165	1.7642
	0.2		4.6860	1.8150	1.2343	0.9424	4.7708	1.8349	1.2477	0.9554
	0.3		3.6952	0.7402	0.2548	0.1136	3.7553	0.7449	0.2560	0.1145
	0.1	2	4.0912	2.4437	1.9923	1.7169	4.1683	2.4806	2.0225	1.7468
	0.2		3.0005	1.5418	1.1517	0.9272	3.0491	1.5606	1.1661	0.9413
	0.3		1.7699	0.4531	0.2085	0.1090	1.7903	0.4556	0.2097	0.1099
	0.1	5	3.2639	2.2348	1.9057	1.6960	3.3388	2.2777	1.9412	1.7295
	0.2		2.0827	1.3532	1.0825	0.9117	2.1220	1.3753	1.0999	0.9280
	0.3		0.5472	0.2683	0.1683	0.1042	0.5510	0.2699	0.1695	0.1053
PFG-2	0.1	0	12.8611	4.5700	3.1098	2.4040	13.1747	4.6312	3.1499	2.4418
	0.2		12.6628	4.3429	2.8910	2.2018	12.9764	4.4015	2.9293	2.2386
	0.3		12.4642	4.1131	2.6698	1.9983	12.7781	4.1693	2.7063	2.0349
	0.1	1	6.2750	3.3568	2.7023	2.3230	6.4032	3.4074	2.7438	2.3652
	0.2		6.0078	3.1316	2.4920	2.1229	6.1319	3.1795	2.5315	2.1642
	0.3		5.7323	2.9007	2.2788	1.9214	5.8519	2.9458	2.3167	1.9627
	0.1	2	4.7929	3.0723	2.5977	2.3006	4.8916	3.1250	2.6428	2.3460
	0.2		4.4716	2.8415	2.3869	2.1005	4.5639	2.8914	2.4303	2.1452
	0.3		4.1302	2.6019	2.1724	1.8988	4.2152	2.6487	2.2142	1.9437
	0.1	5	3.9796	2.8497	2.5011	2.2763	4.0799	2.9108	2.5537	2.3460
	0.2		3.6406	2.6235	2.2928	2.0764	3.7341	2.6827	2.3443	2.1452
	0.3		3.2657	2.3872	2.0804	1.8746	3.3506	2.4444	2.1314	1.9437
PFG-3	0.1	0	12.4842	4.1197	2.6768	2.0092	12.7794	4.1697	2.7068	2.0356
	0.2		11.9088	3.4173	2.0008	1.4057	12.1858	3.4542	2.0196	1.4208
	0.3		11.3334	2.6738	1.2755	0.7884	11.5923	2.6982	1.2844	0.7942
	0.1	1	5.8947	2.9481	2.3004	1.9363	6.0077	2.9872	2.3306	1.9656
	0.2		5.2464	2.3002	1.6765	1.3458	5.3408	2.3263	1.6946	1.3624
	0.3		4.5889	1.6200	1.0243	0.7465	4.6653	1.6344	1.0322	0.7527
	0.1	2	4.4126	2.6750	2.2035	1.9163	4.4961	2.7150	2.2360	1.9478
	0.2		3.7095	2.0385	1.5906	1.3291	3.7728	2.0643	1.6099	1.3470
	0.3		2.9843	1.3768	0.9562	0.7344	3.0284	1.3901	0.9645	0.7411
	0.1	5	3.6026	2.4606	2.1136	1.8948	3.6846	2.5061	2.1511	1.9297
	0.2		2.8850	1.8397	1.5123	1.3113	2.9434	1.8687	1.5346	1.3312
	0.3		2.1286	1.1977	0.8944	0.7216	2.1644	1.2122	0.9038	0.7291

The newly proposed mixed series solution effectively facilitates the analysis of free vibration and buckling in both non-porous and porous 2D-FGBs under various boundary conditions.

- As the values of p_x and p_z increase, the non-dimensional fundamental frequencies and critical buckling loads tend to decrease.
- While the influence of e on $\bar{\omega}$ varies depending on p_x , p_z , and porosity type, it becomes more pronounced as p_x and p_z increase.
- As the material tends towards a more metallic state, it becomes evident that PFG-1 is the most influential porosity type when considering changes in fundamental frequencies and critical buckling loads.
- The total void volume and the void distribution within the cross-section are critical factors influencing the fundamental frequencies and critical buckling loads.
- With an increase in e , the critical buckling loads exhibit an upward trend across all porosity types.
- While the distribution of the vibration and buckling mode shapes remains consistent, there are variations in both the magnitude and the position of the maximum values.
- An increase in the slenderness ratio (L/h) leads to a rise in non-dimensional fundamental frequencies and critical buckling loads.

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Data availability No datasets were generated or analysed during the current study.

Declarations

Conflict of interest The author declares that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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